Neural Networks for Predicting Options Volatility

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Abstract: In this paper, we compare existing methods of estimating the volatility of daily S&P 100 Index for options. The implied volatility, calculated via the Black-Scholes model, is currently the most popular method of estimating volatility and is used by traders in the pricing of options. Historical volatility has been used to predict the implied volatility, but the estimates are poor predictors. A neural network for predicting volatility is shown to be far superior to the historical method.

1. INTRODUCTION

The desire to forecast volatility of financial markets has motivated a large body of research during the past decade (Engle and Rothschild, 1992). Volatility is a measure of price movement often used to ascertain risk. Relationships between volatility and numerous other variables have been studied in an attempt to understand the underlying process so that accurate predictions can be made. The ability to accurately forecast volatility gives the trader a significant advantage in determining options premiums.

Both reseachers and traders use two estimates of option volatility: the historical volatility and the implied volatility. It is almost routinely reported in various publications of exchanges that these two series differ, but no significantly better forecasting model of volatility has emerged. The purpose of this research is to compare these two existing methods of predicting volatility for S&P 100 options with a new approach which uses neural networks. Neural networks, which have been shown to effectively model nonlinear relationships, prove to be a superior approach to predicting options volatility in all cases tested and can be used to develop monthly forecasts.

2. CALCULATING HISTORICAL AND IMPLIED VOLATILITIES

In their seminal work on pricing options, Black and Scholes (1973) assumed that the price of the underlying asset follows an Itô process

$$dS/S=\mu dt+\sigma dZ \tag{1}$$

where dS/S denotes the rate of return, μ is the instantaneous expected rate of return, σ is the expected instantaneous volatility and Z is a standardized Wiener process ,or dZ is a continuous-time random walk. To simplify their analysis, Black and Scholes assumed that both μ and σ were constants and by using an elegant arbitrage argument, they derived their call option pricing model. Their formula expresses the call price C, as a function of five inputs

$$C = C(S, X, T, \sigma, r) \quad (2)$$

where S is the current price of the underlying asset, X is the exercise or strike price, T is the time from now to expiration of the option, σ is the expected instantaneous volatility and r is the riskless short term rate of interest.

Observe that the μ of equation (1) does not appear in (2). The mathematical derivation of the call option pricing formula as shown in Malliaris (1982) shows that arbitrage requires that the per unit of risk excess returns between two appropriately designed portfolios must be equal. Making the necessary substitutions in this arbitrage relationship, the term containing μ drops out. With μ now out of the picture and with four of the five remaining variables directly observable, an estimate of the asset's volatility σ in (2) becomes the focal point of attention for both theorists and traders.

There are two main approaches to estimating and predicting the nonconstant σ: the historical approach and the implied volatility approach. The historical approach is the simplest because tomorrow's volatility σ_{t+1} is an estimate obtained from a sample, of a given size, of past prices of the underlying asset. Suppose that the sample size is n and let

$$S_{t-n+1}, \ldots, S_{t-1}, S_t$$

denote daily historical prices for the underlying asset. To get an estimate for σ_{t+1}, first compute daily returns, r_{t-i}, i=0,...,n-2, where $r_{t-i} = \ln(S_{t-i}) - \ln(S_{t-i-1})$.

For a sample of n historical prices, we obtain (n-1) rates of daily return. The annualized standard deviation of these rates of return is defined as the historical volatility and can be used as an estimate of σ_{t+1}. The nearby historical volatility uses 30 days of data, the middle historical volatility uses 45, and the distant historical volatility has 60 daily prices.

An obvious problem with the historical approach is that it assumes that future volatility will not change and that history will exactly repeat itself. Markets, however, are forward looking and numerous illustrations can be presented to show that historical volatility does not always anticipate future volatility and a better estimate comes from the Black-Scholes option pricing model itself (Choi and Wohar, 1992).

Simply stated, supporters of implied volatility claim that tomorrow's volatility σ_{t+1} can only be estimated during trading tomorrow, i. e., in real time. As option prices are being formed by supply and demand considerations, each trader assesses the asset's volatility prior to making his or her bid or ask prices and, accepting the consensus price of a call as a true market price reflecting the corporate opinions of the trading participants, one solves the Black-Scholes model for the volatility that yields the observed call price. When volatility is calculated in this way, it is called the "implied volatility", with the adjective "implied" referring to the volatility estimate obtained from the Black-Scholes pricing formula. Unlike historical volatility, which is backwards looking to past returns, the implied volatility is forward looking to the stock's future returns from now to the time of the expiration of the option. This implied volatility technique has become the standard method of estimating volatility at the moment of trading.

3. NEURAL NETWORKS FOR PREDICTION

While there are dozens of network paradigms, the backpropagation network has frequently been applied to classification, prediction, and pattern recognition problems. Financial applications of neural networks include underwriting (Collins, Ghosh, and Scofield, 1988), bond-rating (Dutta and Shekhar, 1988), predicting thrift institution failure (Salchenberger, Cinar, and Lash, 1992), and estimating option prices (Malliaris and Salchenberger, 1993). The term backpropagation technically refers to the method used to train the network, although it is commonly used to characterize the network architecture. For details of this method, see Rumelhart and McClelland (1986). Currently, a number of variations on this method exist which overcome some of its limitations.

4. DATA AND METHODOLOGY

Data have been collected for the most successful options market: the S&P 100 (OEX), traded at the Chicago Board Options Exchange. Daily closing call and put prices and the associated exercise prices closest to at-the-money, S&P 100 Index prices, call volume, put volume, call open interest and put open interest were collected from the *Wall Street Journal* for the calendar year 1992.

Three estimates for the historical volatilities using Index price samples of sizes 30, 45 and 60 were computed for each trading day in 1992. We also used the Black-Scholes model to calculate implied volatilities for the closest at-the-money call for three contracts: those expiring in the current month, those expiring one month away, and those expiring two months away (nearby, middle, and distant, respectively). Thus, we have approximately 250 observations for six series of volatilities for use in our study.

Comparisons were made between the nearby historical, implied and network volatility estimates. Because the neural network must have sufficient previous data in order to generalize, these estimates were developed using each method for June 22 through December 30, 1992. Trading cycles were used as the prediction periods, with each trading cycle ending on the third Friday of the month.

5. A COMPARISON OF HISTORICAL AND IMPLIED VOLATILITY ESTIMATES

The historical and implied volatility for the nearby contract are graphed together in Figure 1 for June 22 through December 30, 1992. As can be observed, the historical estimate significantly underestimates the volatility used by most traders, i.e., the implied volatility. Since the historical volatility is an average based on returns from 30 preceding days, it is not surprising that the estimate smoothes out the peaks, giving a value for each day which is less variable, and thus less sensitive to daily market fluctuations. The implied volatility for any given day uses only trading information from that day, not a previous time period, to generate a value. Thus, the implied volatility is more reflective of market changes.

The average MAD (mean absolute deviation) and MSE (mean squared error) for the entire forecasting period, from June 22 through Dec. 30 were 0.0331 and 0.0016. The proportion of times which the historical volatility correctly predicted that the implied volatility would increase or decrease are shown in the last column of the table. An overall average of the number of times a change was correctly indicated is .4439, i.e., a little less than half of the time.

Table 1. A Comparision of Historical and Implied Volatilities

Dates of Forecast	MAD	MSE	Correct Directions
Jun 22--Jul 19	.0318	.0012	8/19 = .421
Jul 20--Aug 21	.0292	.0019	11/25 = .440
Aug 24--Sep 18	.0406	.0018	12/18 = .667
Sep 21--Oct 16	.0479	.0027	7/20 = .350
Oct 19--Nov 20	.0213	.0008	14/25 = .560
Nov 23--Dec 18	.0334	.0014	8/18 = .444
Dec 21--Dec 30	.0294	.0009	2/6 = .333

6. DEVELOPMENT OF THE NEURAL NETWORKS

To develop a neural network which is capable of generalizing a relationship between inputs and outputs, the training set selected must contain a sufficient number of examples which are representative of the process which is being modelled. Therefore, the neural network models developed to predict volatility were trained with data sets from historical data from January 1 through July 18 and used to make predictions for six trading cycles beginning with the period July 20 through August 21 and ending with the period from November 23 through December 31. All prior historical data was used when predicting the volatility for the next trading period. Predicting the volatility for the next cycle is a rather rigorous test of the forecasting capabilities of the network since we are asking it to predict volatility for up to 30 days in the future.

There is no well-defined theory to assist with the selection of input variables and generally, one of two heuristic methods is employed. One approach is to include all the variables in the network and perform an analysis of the connection weights or a sensitivity analysis to determine which may be eliminated without reducing predictive accuracy. An alternative is to begin with a small number of variables and add new variables which improve network performance. In this research, the latter was used and variables were selected using existing financial theory, sensitivity analysis, and correlation analysis. Thus, a number of preliminary models were developed to determine which input variables of the group available in the data set would best predict volatility.

The first models were developed with variables representing volatility lagged from 3 to 7 periods to determine an appropriate set of lag variables. Next, other networks were developed and trained to determine which variables were the best predictors of volatility. The final models include the following 13 variables: change in closing price, days to expiration, change in open put volume, the sum of the at-the-money strike price and market price of the option for both calls and puts for the current trading period and the next trading period, daily closing volatility for current period, daily closing volatility for next trading period, and four lagged volatility variables. By including both the time-dependent path of volatility and related contemporaneous variables in our model, we obtained better predictions.

The backpropagation network developed to predict volatility has 13 input nodes representing the independent variables used for prediction, one middle layer consisting of 9 middle nodes, and an output node representing the volatility. The cumulative Delta Rule for training was selected, with an epoch size of 16, and decreasing learning rate initially set at 0.9 and an increasing momentum, initially set at 0.2. The networks were trained using Neuralworks Professional II software from Neuralware.

7. A COMPARISON OF THE NEURAL NETWORK AND IMPLIED VOLATILITY ESTIMATES

Using historical volatility as a benchmark, we evaluated the performance of the neural network by measuring mean absolute deviation, mean squared error, and the number of times the direction of the volatility (up or down) was corrected predicted. These results are shown in Table 2, where comparisons are made between the volatility forecasted by the network and tomorrow's implied volatility. The overall MAD for the entire period was .0116 and the MSE was .0001 as compared to 0.0331 and 0.0016 when the historical was compared to the implied volatility. Furthermore, for each forecasting period, the MAD and MSE were considerably lower, see Tables 1 and 2. In each of the time periods, the proportion of correct predictions of direction made by the neural network was greater than that of historical volatility. The overall proportion of correct direction predictions was 0.794, as compared to .4439 for the historical volatility estimate. This is not surprising since historical volatility smoothes out the estimate because it is an average of 30 values. The correlation between the implied volatility and the volatility predicted by the network is 0.85, as compared with 0.31 for the historical volatility, at the 5% level of significance..

8. DISCUSSION

The results of this comparative study of neural networks and conventional methods for forecasting volatility are encouraging. Because historical estimates are traditionally poor predictors, traders have been forced to rely on formulas like the Black-Scholes which can be solved implicitly for the real-time volatility. But these models are difficult to use and limited since they can only provide estimates to the traders which are valid at that current time. Furthermore, they fail to incorporate knowledge of the history of volatility. The neural network model, on the other hand, employs both short-term historical data and contemporaneous variables to forecast future implied volatility.

The neural network approach has two advantages which make it more useable as a forecasting tool. First, predictions can be made for a full trading cycle, thus avoiding the problems associated with the need for real-time calculations. Secondly, and more importantly, the network forecasts, in the cases we tested, were very accurate estimates of the volatility preferred by traders.

The limitations of neural networks as financial modelling tools are well-documented. Unlike the more familiar analytical models, a trained neural network does not provide information about the underlying model structure. It is often viewed as a black box since there are no theory-based methods available to interpret and analyze network parameters. Neural networks lack systematic procedures for developing network architecture, selecting training and testing sets, and setting network parameters and thus, are difficult to develop. Explicit knowledge of the phenomenon being predicted is required to assist in variable selection.

There are several ways to extend this research. While the performance of these networks in forecasting volatility is superior to the use of historical volatility, improvement may be possible through experimentation with other variables and network architectures. In this paper, we report results for predicting nearby volatility. However, networks for predicting middle and distant volatility have been developed, using different varialbles and different network architectures.

Table 2. Neural Network and Implied Volatilities

Dates of Forecast	MAD	MSE	Correct Directions
Jun 22--Jul 19	.0148	.0003	16/19 = .842
Jul 20--Aug 21	.0107	.0002	16/25 = .640
Aug 24--Sep 18	.0056	.0001	13/18 = .722
Sep 21--Oct 16	.0127	.0003	19/20 = .950
Oct 19--Nov 20	.0059	.0001	20/25 = .800
Nov 23--Dec 18	.0068	.0001	15/18 = .833
Dec 21--Dec 30	.0039	.0000	5/6 = .833

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