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terrà un seminario dal titolo

"Black-Scholes Options Pricing and Neural Networks"

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Black-Scholes Option Pricing and Neural Networks

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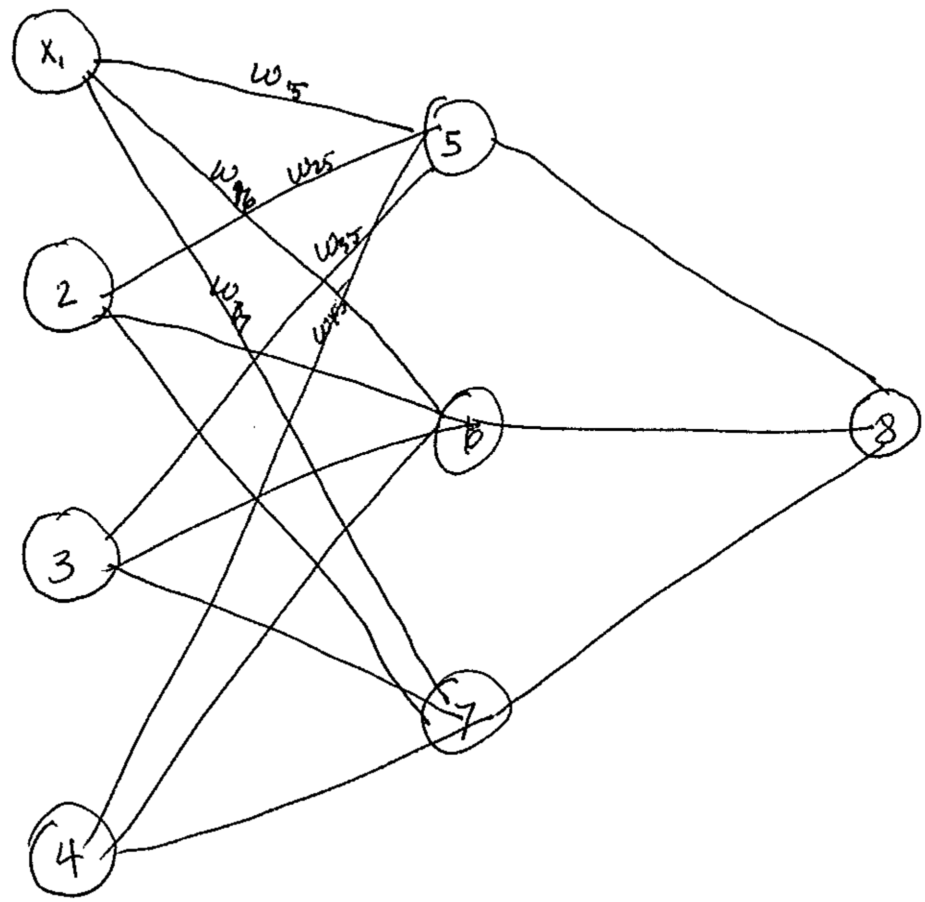
Neural network

the primary elements { neurons, processing elements / connections with weights



learns from examples rather than being told rules or ~~formulas~~

input layer — hidden layer } may be more than 1 — Output layer

feedforward : ~~info~~ data flows in one direction

Supervised learning: answer available in training
unsupervised learning - recognition problems, faces of criminals

backpropagation: an error signal is fed back through the network, altering weights as it goes one node

$$\sum w_K \cdot O_j$$

w_K ↑ ~~each~~ weight

O_j ↑ output from node in previous layer

initial input scaled between 0 and 1.

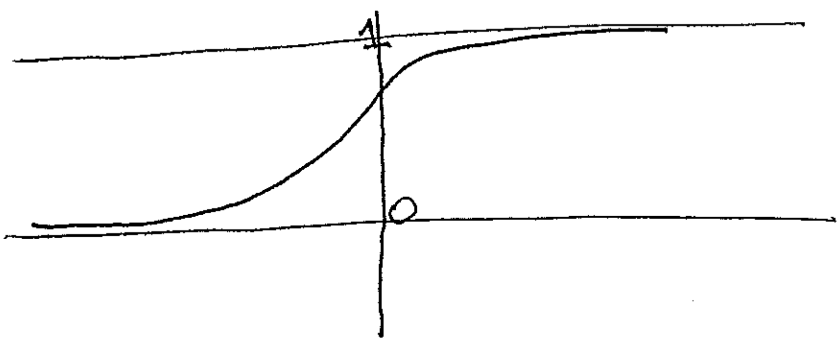
Output goes to many nodes

$$f(\Sigma)$$

↑ threshold activation function

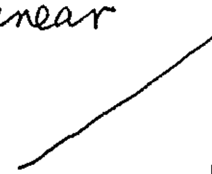
the de facto standard is the sigmoidal or logistic fcn.

$$f(x) = \frac{1}{1+e^{-x}}$$



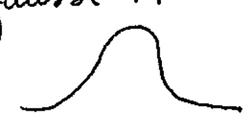
Some other choices:

linear



step

gaussian



Weights are adjusted according to the generalized delta rule

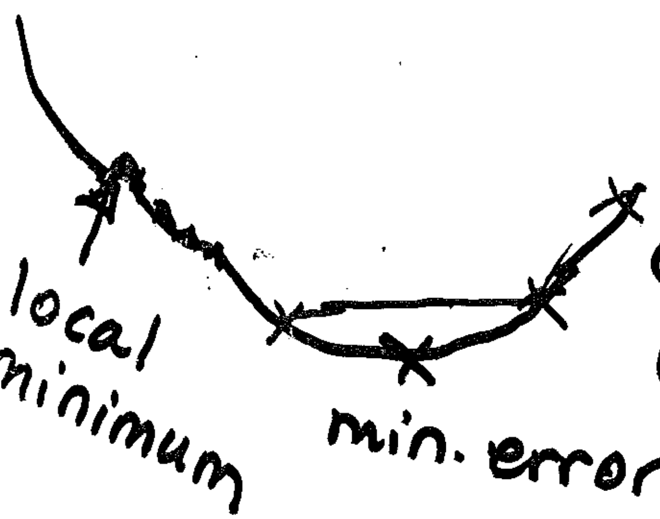
$$MSE = \frac{\sum_{i=1}^{N} (d_i - O_i)^2}{N}$$

$$\Delta W_{ij} = \eta * -\delta E_i / \delta W_{ij}$$

or

$$w_{ij}(t+1) = w_{ij}(t) + \eta \delta_{pi} i_{pj}$$

learning rate
how large the change is to any weight



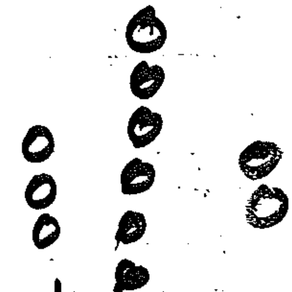
too small: get stuck in local min

default learning rate = 1

What factors need to be adjusted?

inputs

number of layers



number of nodes per layer
(input + output)/2   too many nodes?

learning rate   memorizes
how much weights change   .1 and down

.1 default training tolerance
what is Good? |actual − network| < tol.

training set
1 year? 2 years?   has the relationship changed over time?

forecasting period
1 day

variety
border patterns !!
when do you Retrain?

The network will be presented with a set of examples, and will try to "guess" the right answer.

If the answers are wrong then the weights are adjusted and the examples are fed through again.

This continues until the examples are below a minimum error you have specified. (training tolerance)

Data Preparation

continuous-valued or binary
Dow Jones ; months? Jan: 1 Mar: 3
12 nodes
· changes or actual amounts
imaginary boundaries?

more than one output?

→ networks consider all examples, but only 1 at a time
must add lags of some data
enough examples

good distribution of examples
not all up trends

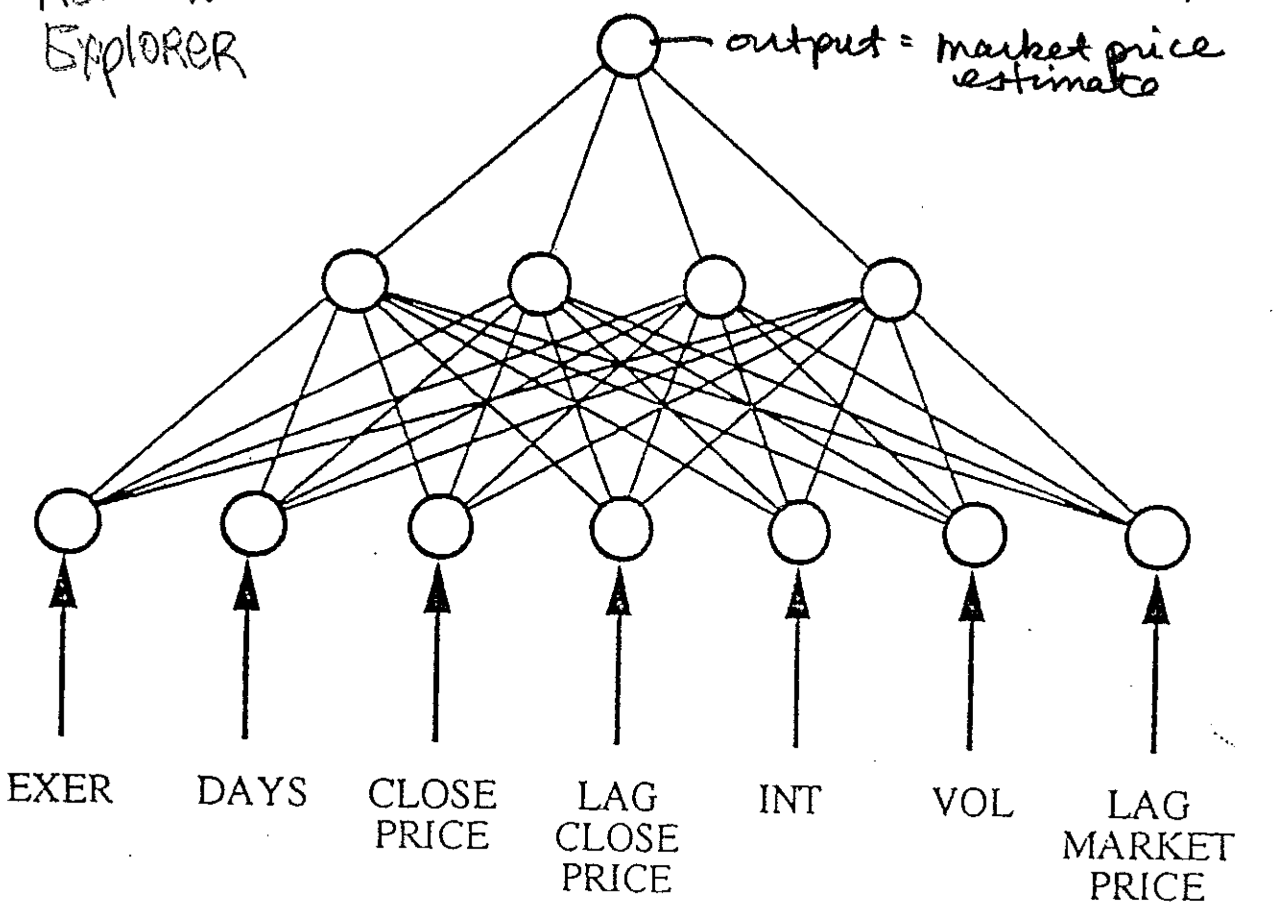
Order to Follow

1. prepare data: in Lotus 1-2-3
2. construct training ← randomize
   (testing
   (forecasting sets
3. decide on network structure + parameters
   #layers #nodes
   learning rate
   tolerance (Good?)
4. run, test, forecast
5. analyze + adapt
   more lags?
   more variables?
   more layers? more nodes?
6. Similarity network?

our network: the 5 B-S variables plus two lagged variables

at first we tried: no lag variables
all variables lagged
2 lags we used had greatest correlation with output

Neuralworks Explorer



What data sets?

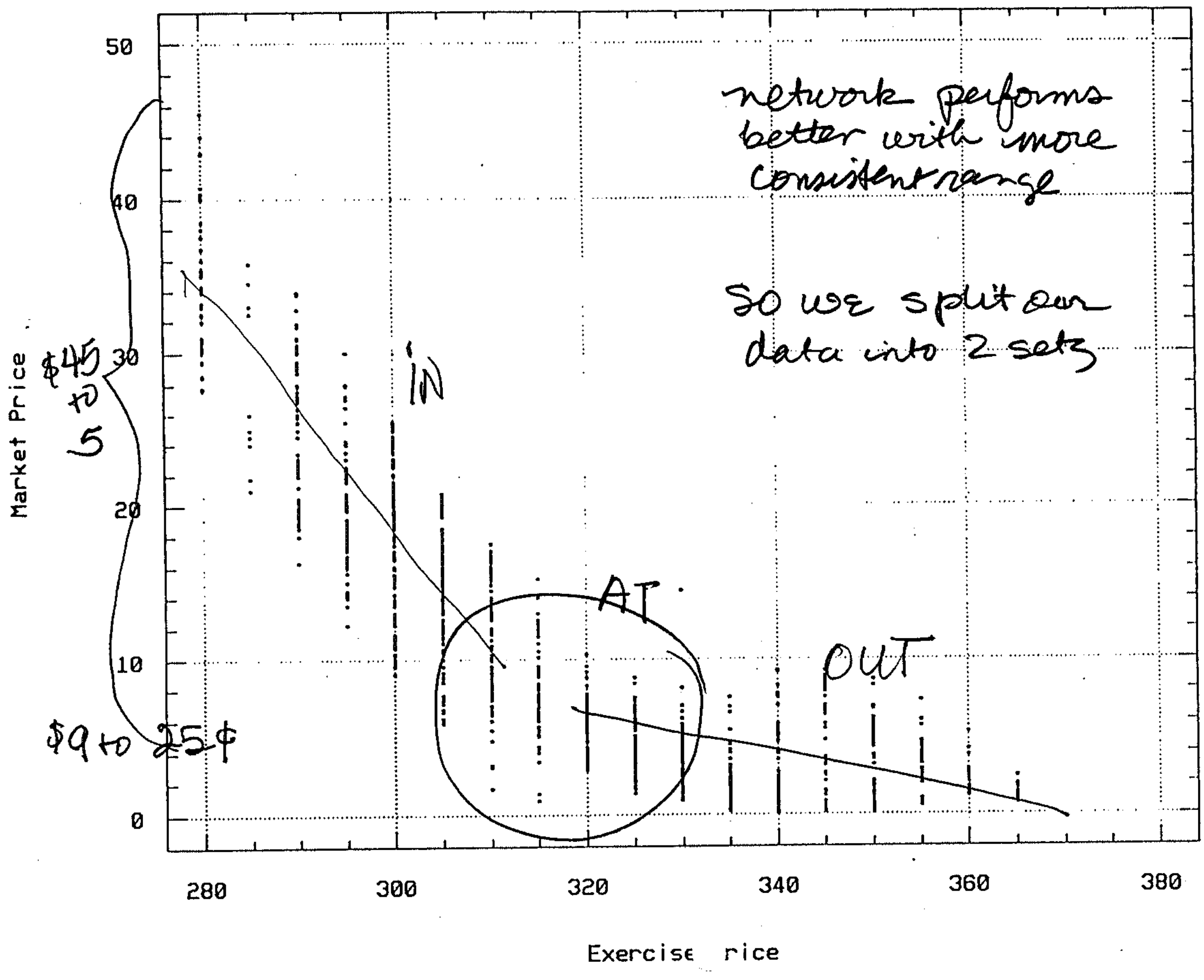


Table 5. In-the-money, paired samples comparison

Better model for IN THE Money for this dataset

Paired Samples Comparison with Black-Scholes

	Black-Scholes	Market Price	Differences
Mean	21.4778	21.58	-0.102209
Variance	104.888	101.118	1.41015
Std. deviation	10.2415	10.0557	1.1875

95% confidence intervals for differences:
Mean: (-0.253529,0.0491108) Accept null hyp
Variance: (1.18749,1.70225)
Std. deviation: (1.08972,1.3047)
Sample size N = 239

reject null hyp of equal means

Paired Samples Comparison with Neural Networks

	Network	Market Price	Differences
Mean	21.0799	21.5785	-0.498506
Variance	95.9599	100.656	1.78591
Std. deviation	9.79591	10.0328	1.33638

95% confidence intervals for differences:
Mean: (-0.668798,-0.328215)
Variance: (1.50392,2.15585)
Std. deviation: (1.22634,1.46828)
Sample size N = 239

not surprising since bias tests indicated tendency of NN to consistently underestimate null hyp of no difference in means is rejected at 05 level for both cases

Table 4. Out-of-the-money, paired samples comparison

Paired Samples Comparison with Black-Scholes

	Black-Scholes	Market Price	Differences
Mean	3.96412	3.45731	0.506307
Variance	5.88913	5.57354	~~0.72131~~
Std. deviation	2.42675	2.36084	0.8493

95% confidence intervals for differences:

Mean:	(0.394979,0.618635) ← overpricing
Variance:	(0.604129,0.876435)
Std. deviation:	(0.777257,0.936181)
Sample size	N = 224

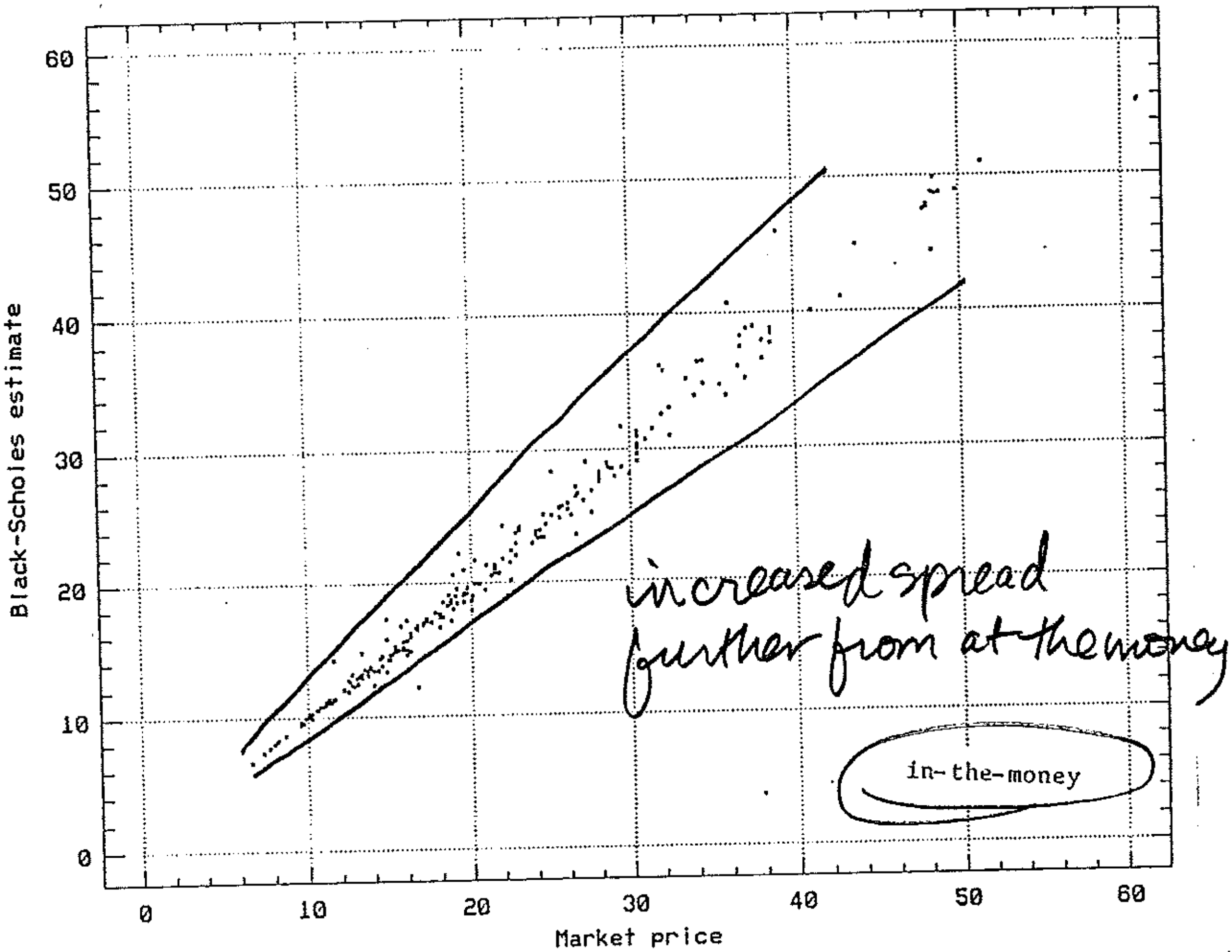
Paired Samples Comparison with Neural Networks

	Network	Market Price	Differences
Mean	3.33894	3.45731	-0.118374
Variance	4.84811	5.57354	~~0.23783~~
Std. deviation	2.20184	2.36084	0.487678

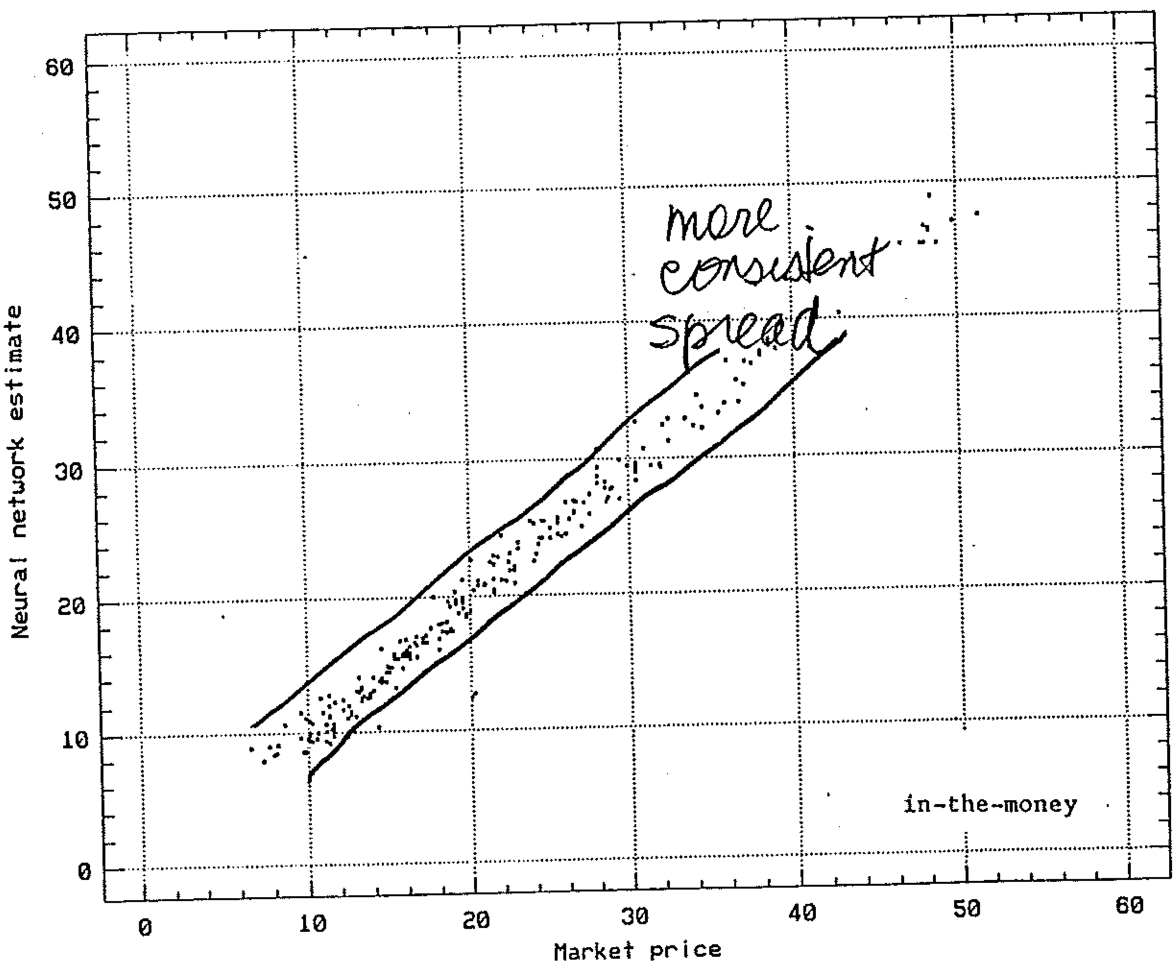
95% confidence intervals for differences:

Mean:	(-0.182587,-0.0541612) ← underpricing
Variance:	(0.199193,0.288978)
Std. deviation:	(0.44631,0.537566)
Sample size	N = 224

Std dev of differences is smaller in the neu. network



Black-Scholes estimate
Market price
in-the-money
increased spread
further from at the money



Neural network estimate
Market price
in-the-money
more
consistent
spread

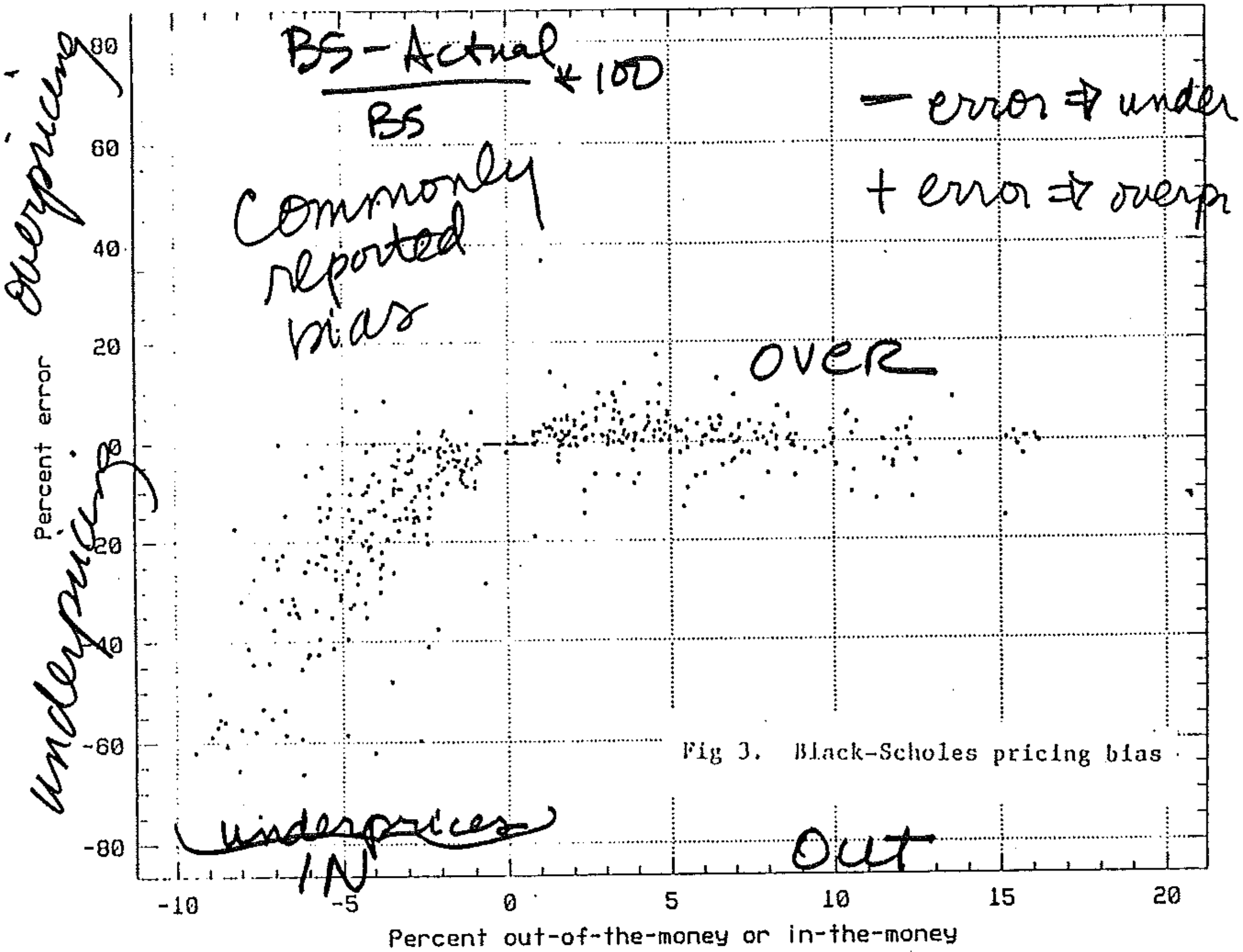


Fig 3. Black-Scholes pricing bias

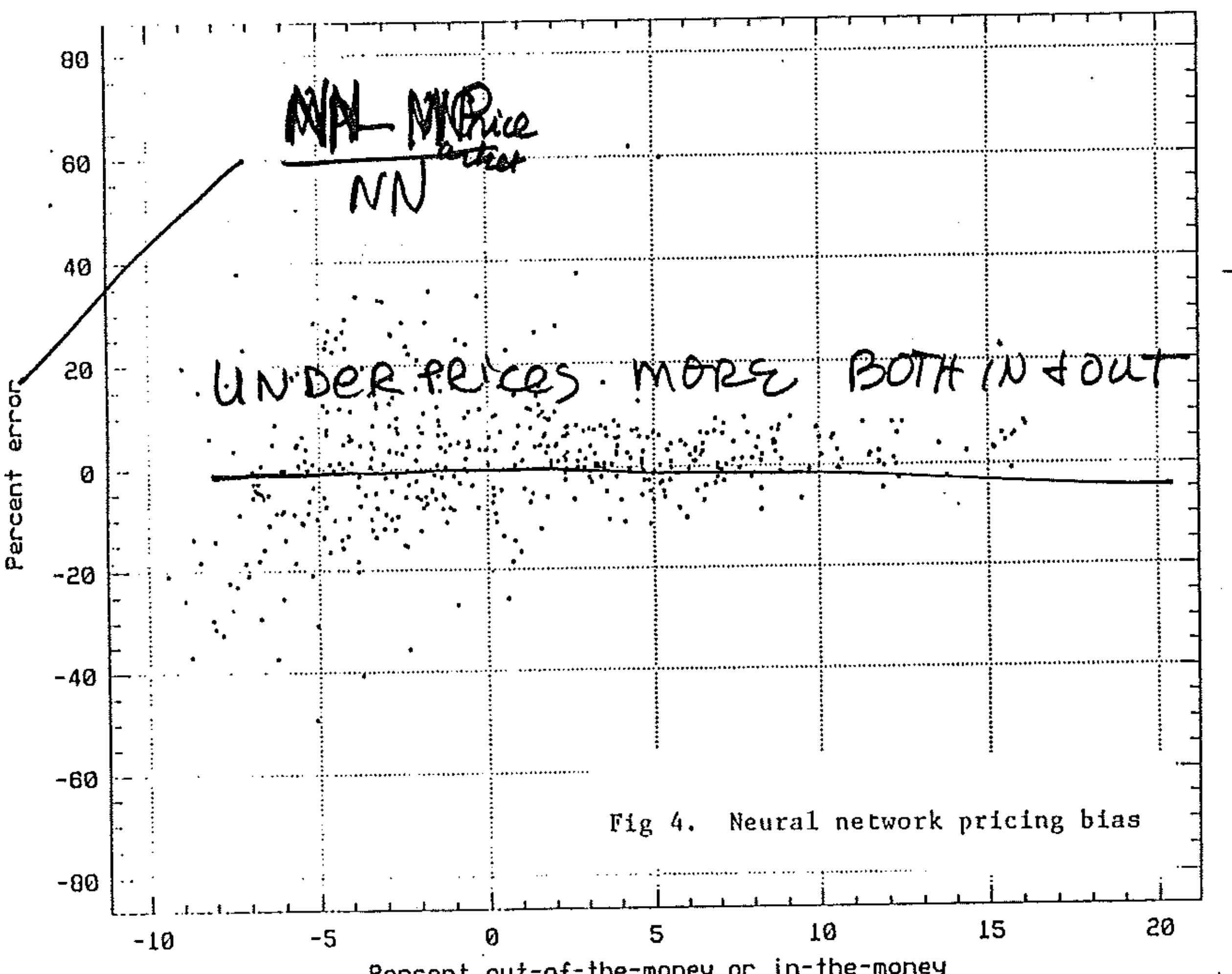


Fig 4. Neural network pricing bias

scatterplots of market price vs. B-S prices