A Neural Network Model for Estimating Option Prices

Mary Malliaris
Management Science Department
Loyola University Chicago
820 N. Michigan Ave.
Chicago, IL 60611
fax: (312) 915-6432
tel: (312) 915-7064

Linda Salchenberger
Management Science Department
Loyola University Chicago
820 N. Michigan Ave.
Chicago, IL 60611
fax: (312) 915-6432
tel: (312) 915-7061

Key words--applied artificial intelligence, neural networks, option pricing, Black-Scholes

Abstract

A neural network model which processes financial input data is developed to estimate the market price of options at closing. The network's ability to estimate closing prices is compared to the Black-Scholes model, the most widely-used model for the pricing of options. Comparisons reveal that the neural network outperforms the Black-Scholes model in about half of the cases examined. The differences and similarities in the two modelling approaches are discussed. The neural network, which uses the same financial data as the Black-Scholes model, requires no distribution assumptions, and learns the relationships between the financial input data and the option price from the historical data. The option valuation equilibrium model of Black-Scholes determines option prices under the assumptions that prices follow a continuous time path and the instantaneous volatility is nonstochastic.

1. Introduction

There has recently been considerable interest in the development of artificial neural networks (ANN's) for solving a variety of problems. Neural networks, which are capable of learning relationships from data, represent a class of robust, nonlinear models inspired by the neural architecture of the brain. Theoretical advances, as well as hardware and software innovations, have overcome past deficiencies in implementing neural networks and made machine learning available to a wide variety of disciplines. Financial applications which require pattern matching, classification, and prediction such as corporate bond rating [1], trend prediction [2], failure prediction [3] and underwriting [4] have proven to be excellent candidates for this new technology.

In this paper, we present a neural network developed to estimate the market prices at closing of OEX options (options on the Standard and Poor's 100) using transactions data for the period January 1, 1990 to June 30, 1990. The neural network is a robust modelling technique which requires no assumptions about price distributions whereas the Black-Scholes model is based on the assumption that prices follow a lognormal distribution. We compare the performance of the neural network and the Black-Scholes option pricing model with actual prices as reported by the CBOE (Chicago Board of Options Exchange) in the Wall Street Journal.

2. Option Pricing Models

In 1973, Black and Scholes [5] proposed a model for computing the current market worth of an option. The discovery of the Black-Scholes model was both empirically and theoretically significant. Its theoretical importance came from finding a solution to a longstanding problem which was initially posed by Louis Bachelier in 1900. He assumed that the price of the underlying asset followed a continuous random walk and proceeded to price the call based on such an asset. The problem with this methodology lies with the fact that with probability 1, the price of the asset becomes negative, which is not consistent with the actual behavior of stock prices, which never take negative values. It took an independent discovery of Ito's calculus in the 1940s and 1950s to develop a mathematical theory for the modeling of continuous time processes. This was used by Black and Scholes in their successful formulation and solution of the option pricing problem.

The empirical significance of the Black-Scholes model lies in its widespread use as a pricing tool on the trading floor. Trading began in 1973, and in less than 20 years, the volume of option trading has increased dramatically. Currently, the trading volume of calls and puts for the OEX is around 760,000 contracts each.

An option is an agreement giving the holder the right to purchase [a call] or sell [a put] some asset at an agreed upon future time, called the date of expiration. European options cannot be exercised before expiration, whereas American options may be exercised at any time prior to expiration. The price that will be paid at this future date is called the exercise price of the option. The market price of the option is the price paid now for the privilege of buying or selling the underlying asset on or before the expiration date. The Black-Scholes model uses five input variables [exercise price of the option, volatility of the underlying asset, price of the underlying asset, number of days until the option expires, and interest rate] to estimate the price which should be charged for an option. The Black-Scholes option pricing formula for calculating the equilibrium price of call options is

$$C=S\bullet N(d_1)-Xe^{-rT}\bullet N(d_2)$$

where C is the market price to be charged for the option, N is the cumulative normal distribution, T is the number of days remaining until expiration of the option expressed as a fraction of a year, S is the price of the underlying asset, r is the risk-free interest rate prevailing at period t, X is the exercise price of the option and d_1 and d_2 are given by

$$d_1=\frac{\ln(\frac{S}{X})+(r+\frac{\sigma^2}{2})\bullet T}{\sigma\sqrt{T}}$$

and

$$d_2 = d_1 - \sigma\sqrt{T}$$

where σ^2 is the variance rate of return for the underlying asset. For any time interval [0,t] of length t, the return on the underlying asset is normally distributed with variance $\sigma^2 t$ [6].

There are seven assumptions underlying this model which assume ideal conditions in the market [5], [7]. They are:

(a) the interest rate is known and constant through time;

(b) the stock price follows a random walk in continuous time with variance proportional to the square of the stock price; thus the distribution of prices is log-normal and the volatility is constant;

(c) the stock pays no dividends;

(d) the option is European, i.e. it can only be exercised at maturity (expiration date);

(e) there are no transactions costs of buying or selling,

(f) the market operates continuously, and

(g) there are no penalties to short selling.

For a rigorous presentation of the derivation of the Black-Scholes model, see [8].

For the remainder of this paper and for the data we have chosen, the exercise price of the option is referred to as exercise price and is denoted by EXER; the annualized square root of the variance of the underlying asset is the volatility, VOL; the price of the underlying asset is the price of the S&P 100 index at closing, or simply, the closing price, CLOSE PRICE ; the interest rate used is the prevailing rate on treasury bills, denoted INT; and time to expiration of the option is the number of days to expiration, DAYS. The exercise price, number of days to expiration, and closing price are observable. The volatility cannot be directly observed so it is computed implicitly. Most observers use the Implied Standard Deviation of observed option prices as an estimate of volatility [9]. We used at-the-money call options for this estimate and then used that estimate of volatility to calculate the call options for that day.

Options with an exercise price equal to the closing price of the index are said to be at-the-money. In the pricing of calls, exercise prices less than the closing price are in-the-money, and exercise prices greater than the closing price are out-of-the-money. If the ratio of the closing price to the exercise price is less than [greater than] .85 [1.15] then the option is said to be deep-out-of- [deep-in-] the-money.

Since its introduction in 1973, the Black-Scholes options pricing model has performed better overall than any model. The major alternative models have been Cox and Ross' pure jump model, Merton's mixed diffusion-jump model (both these models relax the continuous time assumption), Cox and Ross' constant elasticity of variance diffusion model, Geske's compound option diffusion model, and Rubinstein's displaced diffusion model (these last three relax the assumption of constant volatility).

Galai [10] extensively surveyed results from competing models. He found that no alternative model yielded better results on a constant basis than did Black-Scholes, even though the Black-Scholes did not give consistently good estimates for deep-in and deep-out-of-the money options. It performs best when estimating market prices at-the-money.

Chesney & Scott [9] in a test of 5 models, some variations of the Black-Scholes model, found that the Black-Scholes model with the implied standard deviation had a performance superior to all others tested, as measured by the MAD and MSE.

Option trading has also been considered an appropriate domain for expert system applications. A constraint logic programming model [11] has been developed as an expert system which uses the Black-Scholes model to evaluate strategies and compute option values. Constraint satisfaction has been used for other approaches to option price modelling [12]; however, no strong measures of the effectiveness of these models have been reported.

Empirical tests show that Black-Scholes remains superior among option pricing equilibrium models, with the possible exception of cases in which trades are made deep-in and deep-out-of-the-money. The volume of research which continues to proliferate related to the Black-Scholes model, even 20 years after its introduction, indicates there is considerable interest and value in developing a model which is more robust than Black-Scholes. In addition, there is some reason to believe that the trading process itself may reveal underlying strategies as well as analytical models and there is information to be gained from historical pricing data. Neural networks have been shown to be useful in modelling nonstationary processes and nonlinear dependencies and thus, may represent a channel of investigation in the search for another type of option pricing model.

3. Methodology

3.1. The data set

The data set used for this research was developed using option price transactions data published in the Wall Street Journal during the period from January 1, 1990 to June 30, 1990. The data set selected for testing includes pricing data from April 23 to June 29, 1990 and includes in-the-money options and out-of-the-money options with time to expiration between 30 and 60 days. Typically, 6 different call prices per day are quoted.

The five variables selected to estimate the market price of the option (MARKET PRICE) are those used in the Black-Scholes model; exercise price (EXER), time to expiration (DAYS), closing price (CLOSE PRICE), volatility (VOL), and interest rate (INT). The Black-Scholes variables were used because we wanted to compare the relative performances of the two models. A sample data set used to calculate the Black-Scholes model prices is included in Table 1. For the neural network, we added two lagged variables: yesterday's closing price, LAG CLOSE PRICE, and yesterday's market price of the option, LAG MARKET PRICE.

Preliminary data analysis revealed dependencies and relationships between the variables which were used to partition the data sets for the neural network. Figure 1 shows a graph of exercise prices versus market prices. From deep-in-the-money to at-the-money, there is a sharp and steady decrease of prices. From at-the-money through out-of-the-money, the prices have a gentle asymptotic approach to the x-axis. Experimentation with different training sets showed that better results could be obtained in the neural networks when the data was separated into in-the-money and out-of-the-money groups. Prices in-the-money vary from \$60.00 to \$0.75; prices out-of-the-money vary from \$15.50 to \$0.0625. A larger proportion of observations exist for out-of-the-money prices than for in-the-money prices. Correlations were also found between time to expiration and market price of the option, and between the closing price and the market price of the option.

3.2. The Estimation Process

Under supervised learning, the feedforward, backpropagation neural network learns relationships between input and output variables during a training process, as data are presented to the network. One approach to testing the performance of the network is to check its accuracy in estimating values for a holdout sample generated from the training set. For evaluating the performance of the option price neural network, we selected a more realistic and more difficult performance measure. The network was trained using historical data and option price estimations for a future period were developed with the trained network and compared to actual prices.

To capture the volatile nature of the options market, a relatively short time frame was used for the training sets and testing sets. The testing sets were developed using a two-week time frame; this was a convenient choice because interest rate and volatility changed weekly and were relatively stable over a two-week period. Five two-week periods were selected for price estimation; the weeks beginning April 23, May 7, May 21, June 4, and June 18. To provide the neural network models with a variety of examples, each training set included as many observations as necessary to provide at least one full cycle (30 days prior to the estimation period) of pricing data.

4. The Neural Network Model for Option Pricing

4.1. Neural Networks and Backpropagation

Inspired by studies of the brain and the nervous system, neural networks are composed of neurons or processing elements and connections, organized in layers. These layers can be structured hierarchically, and the first layer is called the input layer, the last layer is the output layer, and the interior layers are called the middle or hidden layers. Feedforward networks map inputs into outputs with signals flowing in one direction only, from the input layer to the output layer. Each connection between neurons has a numerical weight associated with it which models the influence of an input cell on an output cell. Positive weights indicate reinforcement; negative weights are associated with inhibition. With supervised learning, connection weights are learned by the network through a training process, as examples from a training set are presented repeatedly to the network.

Each processing element has an activation level, specified by continuous or discrete values. If the neuron is in the input layer, its activation level is determined in response to input signals it receives from the environment. For cells in the middle or output layers, the activation level is computed as a function of the activation levels on the cells connected to it and the associated connection weights. This function is called the transfer function or activation function and may be a linear discriminant function, i.e., a positive signal is output if the value of this function exceeds a threshold level, and 0 otherwise. It may also be a continuous, nondecreasing function. The most commonly used for backpropagation is the sigmoidal or logistic function

$$f(x)=\frac{1}{1+e^{-\gamma x}}$$

where γ is a constant which controls the slope.

While basically an information processing technology, neural networks differ from traditional modelling techniques in a fundamental way. Parametric models require that the developer specify the nature of the functional relationship between the dependent variable and the independent variables e.g., linear, logistic. Neural networks with at least one middle layer use the data to develop an internal representation of the relationship between the variables so that a priori assumptions about underlying parameter distributions are not required. As a consequence, we might expect better results with neural networks when the relationship between the variables does not fit the assumed model. Nevertheless, many decisions regarding model parameters and network topology can affect the performance of the network.

Two-layer neural networks do not have the ability to develop internal representations. They map input patterns into similar output patterns. While these networks have proved useful in a variety of applications, they cannot generalize or perform well on patterns which have never been presented. A two-layer feedforward backpropagation neural network can be developed which is similar in structure to the familiar linear regression model [13].

In a feedforward neural network, the connection weights can be determined during a two-step training process that presents examples $\{(x_p, y_p): p = 1,...,P\}$ where x_p is the input vector and y_p is the output vector. In the first step, for each layer of nodes, the network computes the output vector o_p as a function of the input vector and the associated connection weights. The values for the output layer nodes are compared to the actual output vector and a performance criteria, like the sum of the squared error, is used to determine the error for the output layer. In the second step, the error is backpropagated through the network and the weights w_{ij} are modified, according to their contribution to the network error F. For further details, see [14].

4.2. Assumptions for the Neural Network

Since the purpose of our study is to compare the call option price estimations made by Black-Scholes with prices estimated by a neural network model, many of the fundamental assumptions made by Black and Scholes in their option pricing model are included as required assumptions for the network as well. Assumptions (a), (c) - (g) will hold true for the neural network, while (b), which requires that the distribution of the prices be lognormal, may be relaxed. The input nodes of the neural network represent the same 5 input variables used to generate the Black-Scholes price estimations. The variable volatility (VOL) for the neural network is measured in the same way as it is for the Black-Scholes model.

4.3. The Development of the Neural Network for Option Pricing

Since feedforward, single hidden layer neural networks have been successfully used for classification and prediction, we selected this network model for our initial experiments and used the backpropagation training algorithm. A neural network consisting of 7 input nodes, 4 middle layer nodes, and 1 output node was developed (see Figure 2). The input nodes represent the five financial variables used in the Black-Scholes model (EXER, DAYS, CLOSE PRICE, VOL, and INT) and two lag variables (LAG CLOSE PRICE and LAG MARKET PRICE), and the output node (MARKET PRICE) represents the market price of the option. Determining the proper number of nodes for the middle layer is more of an art than a science and experimentation and heuristics assisted in making this choice. Generally speaking, too many nodes in the middle layer, and hence, too many connections, produce a neural network which memorizes the input data and lacks the ability to generalize. Therefore, increasing the number of nodes in the middle layer will improve performance on the training set while decreasing the number of nodes in the middle layer will improve performance on a new data set. This proved to be true for our application and 4 middle layer nodes gave the best results.

The network is fully connected, with a direct connection from exercise price (EXER) to the output node (MARKET PRICE). Better results were achieved with this additional connection because of the linear dependence between EXER and MARKET PRICE observed in the data set and verified with a series of regression models. All the connection weights were initially randomized, and were then determined during the training process.

The generalized Delta rule was used with the backpropagation of error to transfer values from internal nodes. (For a more detailed explanation of backpropagation learning and the generalized Delta rule, see [14].) The sigmoidal function is the activation function specified in this neural network and is used to adjust weights associated with each input node.

Supervised learning was conducted with training sets consisting of the seven predictor variables and the corresponding market price of the option for each exercise price, for each trading day. For the input nodes in which the data was not in ratio form, the values were scaled to be within a range of 0 to 1. This minimizes the effect of magnitude among the inputs and increases the effectiveness of the learning algorithm. The selection of the examples for the training set focused on quality and the degree to which the data set represented the population. The size of the training set is important since a larger training set may take longer to process computationally, but it may accelerate the rate of learning and reduce the number of iterations required for convergence.

The learning rate and momentum were set initially at 0.9 and 0.6, respectively and the learning rate was adjusted downward and the momentum was adjusted upward to improve performance. The training examples were presented to the network in random order to maximize performance and to minimize the introduction of bias. Training was halted after a minimum of 40,000 iterations. The network was implemented using the software package Neuralworks Professional II Plus® running on a 386-based microcomputer with a math co-processor.

4.4. Experimental Design

To compare the estimations made by each model, we compute and report the mean absolute deviation (MAD), mean absolute percent error (MAPE), and mean squared error (MSE) for each of the 5 two-week periods for both in-the-money and out-of-the-money prices. While MAD and MSE are meaningful measures of error for this application, we were most concerned with MAPE. Since prices vary from $60.00 to $0.0625, it is important to compare the amount of the error with the corresponding base price, i.e., to measure the relative pricing error. Option prices were estimated from the Black-Scholes model using a computer program based on equations (1)-(3). Neural network estimations were developed by inputting the estimation sets into a trained network.

5. Results

The initial results showed that, compared to the actual prices, the neural network estimations had a lower MAPE than Black-Scholes for 4 of the 5 two-week periods for the out-of-the-money case, but Black-Scholes was superior for 4 of 5 two-week periods for in-the-money trades. These results are reported in Tables 2 and 3. A bias commonly reported in the literature is that Black-Scholes tends to underprice in-the-money calls [9]. To examine pricing bias, we plot the percent pricing error versus the percent the option is in-the-money or out-of-the-money. Pricing error is calculated as the difference in the model price and actual market price, divided by the model price. Pricing error is negative when the model underestimates the actual market price and is positive when the model overestimates the market price. The percent in-the-money or out-of-the-money is found by calculating the difference in the exercise price and the closing price and then dividing by the exercise price.

Pricing bias was investigated for both the Black-Scholes and the neural network models. Figures 3 and 4 show percent error (values greater than 0 indicate overpricing and less than 0 indicate underpricing) relative to percent in or out-of-the-money (negative values indicate out-of-the-money, positive are in-the-money). In the Black-Scholes model (see Figure 3), underpricing is more prevalent than overpricing for in-the-money and overpricing is predominant for out-of-the-money. The neural network model (see Figure 4) underprices options more than it overprices them, for both in and out-of-the-money. For both models, the most serious errors occur out-of-the-money; however, the overpricing errors are more significant for the Black-Scholes model as prices move deep out-of-the-money.

Paired sample comparisons tests were run on the Black-Scholes estimates and actual market prices and on the neural network estimates and actual market prices. In Table 4, we report the results for out-of-the-money prices. The means, variances and standard deviations for each sample and for the differences between the model price and the actual price are reported. The null hypothesis of no difference in the means is rejected at the 5% significance level for each model. The 95% confidence intervals for the mean differences show that the Black-Scholes consistently overprices the options, while the neural network underprices them. We also observe that the standard deviation of the differences is smaller in the neural network prices.

Results of the paired sample comparisons test for the in-the-money cases are shown in Table 5. There is a statistically significant difference between the means of the sample of neural network predictions and the sample of actual market prices. This is not surprising since the bias tests indicated the tendency of the neural network to consistently underestimate prices. The Black-Scholes however, did not show a significant difference from zero, hence it provides a better model for in-the-money, for this data set.

Scatterplots were developed showing the market prices versus the Black-Scholes prices (see Figures 5 and 7) and the market prices versus neural network predictions (see Figures 6 and 8). From Figures 5 and 6, which show out-of-the-money prices, we observe more outliers in the Black-Scholes estimates than in the neural network estimates. This is consistent with the higher standard deviation found in the paired comparisons test. For the neural network estimates, prices furthest from at-the-money are more clustered than for the Black-Scholes. While a strong linear relationship is indicated in each, more variation is observed in the Black-Scholes as the market prices become larger, i.e., as prices move further from at-the-money. Figures 7 and 8, which show in-the-money prices, show more consistent spread for the neural network prices while the Black-Scholes prices are more clustered near at-the-money, which is the expected result.

A few observations about the results can be made. First, although we have only presented summary statistics, one can observe similarities between the individual price estimates made by the two models. Each model has difficulty computing prices when the trades are deep in-the-money. This is expected for the neural network because the majority of trades are close to at-the-money and thus, there are insufficient examples to present to the network for these cases. Secondly, we would not expect to achieve results with the neural network which are significantly different than those of Black-Scholes if many traders are using the Black-Scholes model and the market prices reflect their strategies. The neural network is only capable of learning the relationships which are imbedded in the observations. The neural network exhibited a bias of underpricing the options and in fact, may be best utilized as input into another pricing mechanism. For the 10 weeks beginning April 23, the neural network outputs were highly correlated with the actual prices and a simple regression equation with the neural network outputs as the price predictor variable was observed to perform well. (This approach has been successfully used in other applications, e.g. [2]).

6.1 Summary and Conclusions

This empirical examination of the Black-Scholes option valuation model and the neural network option pricing model leads to some interesting conclusions. While both models perform best when estimating prices close to at-the-money, the Black-Scholes makes greater overpricing errors deep-out-of-the-money, showing many more outliers. A common result emerges for in-the-money cases, with both models consistently underpricing options. However, for both in- and out-of the money prices, the neural network outperforms the Black-Scholes model in about 50 percent of the cases examined.

Our results demonstrate that the neural network methodology offers a valuable alternative to estimating option prices to the traditional Black-Scholes model. The evidence reported here is encouraging, particularly in view of the essentially undisputed superiority of the Black-Scholes model. Analytically, it is interesting that the well-developed methodology of Black-Scholes, with its explicit formula for pricing options, derived using sophisticated financial arbitrage arguments and advanced stochastic calculus techniques, can actually be approximated by neural networks.

There are several limitations which may restrict the use of neural network models for estimation. There is no formal theory for determining optimal network topology and therefore, decisions like the appropriate number of layers and middle layer nodes must be determined using experimentation. The development and interpretation of neural network models requires more expertise from the user than traditional analytical models. Training a neural network can be computationally intensive and the results are sensitive to the selection of learning parameters, activation function, topology of the network, and the composition of the data set.

REFERENCES

1. Dutta, S and Shekhar, S (1988) Bond-rating: A non-conservative application of neural networks. Proc. of the IEEE International Conference on Neural Networks **2**, 142-150.

2. Jang G, Lai F, Jiang B, and Chien L (1991) An intelligent trend prediction and reversal recognition system using dual-module neural networks. Proc. of the First International Conference on Artificial Intelligence Applications on Wall Street, IEEE Computer Press, 42-51.

3. Salchenberger L, Cinar E M, and Lash, N (1992) Neural networks: A new tool for predicting thrift failures. Dec. Sci., 23(4), 899-916.

4. Collins E, Ghosh S, and Scofield C (1988) An application of a multiple neural-network learning system to emulation of mortgage underwriting judgments. Proc. of the IEEE International Conference on Neural Networks **2**, 459-466.

5. Black F and Scholes M (1973) The pricing of options and corporate liabilities. J. Pol. Econ. **81**, 637-654.

6. Macbeth J and Merville L (1979) An empirical examination of

the Black-Scholes call option pricing model. J. Fin. **34**(5), 1173-1186.

7. Chiras D and Manaster S (1978) The information content of option prices and a test of market efficiency. J. of Finl. Econ. **6**, 213-234.

8. Malliaris A and Brock W (1982) Stochastic Methods in Economics and Finance, North-Holland, Amsterdam.

9. Chesney M and Scott L (1989) Pricing European currency options: a comparison of the modified Black-Scholes model and a random variance model. J. Fin. Quant. Anal. **24**, 267-284.

10. Galai, D (1983) A survey of empirical tests of option-pricing models. In Option Pricing (Edited by Brenner M). Lexington Books.

11. Yap R (1987) Constraint logic programming and option trading. IEEE Expert, 42-50.

12. Mizogushi G and H Ohwada L (1991) A constraint-oriented decision support system for option-based investment

strategies. Proc. First International Conference on Artificial Intelligence Applications on Wall Street, IEEE Computer Press, 160-167.

13. White H (1989) Some asymptotic results for learning in single hidden layer feedforward network models. J. Am. Statist. Assoc. **84**, 1003-1013.

14. Rumelhart D E and McClelland J L (1986) Parallel Distributed Processing. MIT Press, Cambridge, MA.

Table 1. Sample data set

DATE M	D	Y	EXER	DAYS	CLOSE PRICE	INT RATE	VOL	MARKET PRICE
4	23	90	280	26	315.58	7.71	0.161633	39.5
4	23	90	290	26	315.58	7.71	0.161633	28
4	23	90	295	26	315.58	7.71	0.161633	22.5
4	23	90	300	26	315.58	7.71	0.161633	17.875
4	23	90	305	26	315.58	7.71	0.161633	14.5
4	23	90	310	26	315.58	7.71	0.161633	10
4	23	90	315	26	315.58	7.71	0.161633	6.625
4	24	90	280	25	313.96	7.77	0.166284	38
4	24	90	290	25	313.96	7.77	0.166284	27.25
4	24	90	295	25	313.96	7.77	0.166284	22.5
4	24	90	300	25	313.96	7.77	0.166284	17
4	24	90	305	25	313.96	7.77	0.166284	12.75
4	24	90	310	25	313.96	7.77	0.166284	8.875
4	25	90	280	24	315.06	7.77	0.159941	36
4	25	90	300	24	315.06	7.77	0.159941	17
4	25	90	305	24	315.06	7.77	0.159941	13.5
4	25	90	310	24	315.06	7.77	0.159941	9.125
4	25	90	315	24	315.06	7.77	0.159941	6
4	26	90	280	23	315.82	7.77	0.158642	37
4	26	90	290	23	315.82	7.77	0.158642	25.375
4	26	90	295	23	315.82	7.77	0.158642	21.5
4	26	90	300	23	315.82	7.77	0.158642	17.625
4	26	90	305	23	315.82	7.77	0.158642	13.5
4	26	90	310	23	315.82	7.77	0.158642	9.5
4	26	90	315	23	315.82	7.77	0.158642	6.25
4	27	90	280	22	312.48	7.77	0.136054	32.375
4	27	90	290	22	312.48	7.77	0.136054	23
4	27	90	295	22	312.48	7.77	0.136054	18.5
4	27	90	300	22	312.48	7.77	0.136054	14.25
4	27	90	305	22	312.48	7.77	0.136054	10
4	27	90	310	22	312.48	7.77	0.136054	6.375

Table 2. Comparative analysis, actual prices with estimated prices, out-of-the-money

Week beginning	MAD	MAPE	MSE
April 23			
Black-Scholes	0.598932	30.81731	0.435342
Neural Network	0.207702	12.74440	0.074409
May 7			
Black-Scholes	0.340729	16.23661	0.160047
Neural Network	0.382937	15.04892	0.253373
May 21			
Black-Scholes	0.378636	9.43207	0.204219
Neural Network	0.422369	12.30240	0.253676
June 4			
Black-Scholes	0.286645	9.104615	0.245477
Neural Network	0.312945	9.097162	0.231779
June 18			
Black-Scholes	0.660788	17.45452	1.250466
Neural Network	0.447812	10.94668	0.455352

Table 3. Comparative analysis, actual prices with estimated prices, in-the-money

Week beginning	MAD	MAPE	MSE
April 23			
Black-Scholes	0.676936	3.8057	1.055732
Neural Network	0.82434	5.1689	1.175115
May 7			
Black-Scholes	0.670291	2.7142	1.459734
Neural Network	1.289340	7.4727	3.127410
May 21			
Black-Scholes	0.766019	2.8867	1.386018
Neural Network	0.832762	4.6876	1.006885
June 4			
Black-Scholes	0.784969	2.8864	1.771367
Neural Network	1.053282	5.2136	2.397112
June 18			
Black-Scholes	1.391258	7.2002	3.945318
Neural Network	0.987918	6.6399	1.407175

Table 4. Out-of-the-money, paired samples comparison

Paired Samples Comparison with Black-Scholes

	Black-Scholes	Market Price	Differences
Mean	3.96412	3.45731	0.506807
Variance	5.88913	5.57354	0.72131
Std. deviation	2.42675	2.36084	0.8493

95% confidence intervals for differences:

Mean:	(0.394979,0.618635)
Variance:	(0.604129,0.876435)
Std. deviation:	(0.777257,0.936181)
Sample size	N = 224

Paired Samples Comparison with Neural Networks

	Network	Market Price	Differences
Mean	3.33894	3.45731	-0.118374
Variance	4.84811	5.57354	0.23783
Std. deviation	2.20184	2.36084	0.487678

95% confidence intervals for differences:

Mean:	(-0.182587,-0.0541612)
Variance:	(0.199193,0.288978)
Std. deviation:	(0.44631,0.537566)
Sample size	N = 224

Table 5. In-the-money, paired samples comparison

Paired Samples Comparison with Black-Scholes

	Black-Scholes	Market Price	Differences
Mean	21.4778	21.58	-0.102209
Variance	104.888	101.118	1.41015
Std. deviation	10.2415	10.0557	1.1875

95% confidence intervals for differences:

- Mean: (-0.253529,0.0491108)
- Variance: (1.18749,1.70225)
- Std. deviation: (1.08972,1.3047)

Sample size N = 239

Paired Samples Comparison with Neural Networks

	Network	Market Price	Differences
Mean	21.0799	21.5785	-0.498506
Variance	95.9599	100.656	1.78591
Std. deviation	9.79591	10.0328	1.33638

95% confidence intervals for differences:

- Mean: (-0.668798,-0.328215)
- Variance: (1.50392,2.15585)
- Std. deviation: (1.22634,1.46828)

Sample size N = 239