A. G. MALLIARIS - M. E. MALLIARIS

A MODEL WITH REAL CASH BALANCES IN PRODUCTION AND UTILITY FUNCTIONS

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A MODEL WITH REAL CASH BALANCES IN PRODUCTION AND UTILITY FUNCTIONS

by

A. G. Malliaris and M. E. Malliaris (*)

Summary

This paper develops, solves and discusses a static general equilibrium model with real cash balances in production and utility functions in order to study problems of integrating monetary and value theory. Under the assumption of profit maximization, the model yields not only input demand functions for capital and labor but also a demand function for money viewed as a producer good. Furthermore under the assumption of constrained utility maximization, the model yields demand functions not only for output but also for real cash balances viewed as a consumer good. Thus the introduction of money in a barter economy is witnessed by the way money and monetary constants are present in most equations of the model. Also a monetary market is developed which allocates real cash balances as producer and consumer goods.

I. *Introduction.*

It is the purpose of this paper to develop, solve and discuss a static general equilibrium model with real cash balances. Such a model is hoped to provide some useful insights to the economists' endeavor to integrate monetary theory and value theory (1).

(*) Assistant Professor of Economics, Loyola University of Chicago and Graduate Student, Northwestern University, respectively.

(1) Integration of monetary and value theory is a vast area of study in economics. For a brief but brilliant study of the issues involved, the reader is referred to H. G. Johnson, « Monetary Theory and Policy », *American Economic Review*, Vol. 52 (June 1962), pp. 335-384.

The intellectual parenthood of the present study is the work of Patinkin ([2]). Patinkin has postulated both in his book as well as elsewhere ([3]) that it is theoretically justified to view real cash balances both as a consumer good yielding utility to the holder, and as a producer good, yielding productive services to the producer. Such an approach has been adopted by several monetary economists ([4]). It is true however, that certain conceptual difficulties are attached to such an approach and that « monetary theory still lacks a Monetary Debreu » ([5]).

Consider a barter economy with three consumption goods and two inputs, i.e., capital and labor. Such an economy suffers from economic inefficiencies in both the production and exchange of goods. Although such an economy is not as complex as our modern economies, the « double coincidence » of wants does not take place very naturally. The introduction of money facilitates exchange and contributes to a more efficient production. The economic theorist immediately would raise the question: How can we theoretically explain the introduction of money in a barter economy and how can we prove that money affects the solutions of a barter model?

In what follows, it will be demonstrated that if money is viewed as both a consumer and producer good then money in the form of real cash balances can be introduced as an argument in the production and utility functions of a mathematical model and its effects could be traced by the development and solutions of the model. Since we are interested in a demonstration, rather than an abstract proof, a solvable Cobb-Douglas general equilibrium model is chosen. A static analysis, rather than a dynamic or even a stochastic analysis, is followed due to its simplicity. The notation of the model is presented in section II. The model is described in section III. Finally sections IV and V are referred to the solutions of the model and some interpreting remarks, respectively.

([2]) D. Patinkin, *Money, Interest and Prices*, 2nd ed., New York, 1965.

([3]) For a more recent methodological reformulation of Patinkin's ideas as they apply within the context of economic growth, see D. Levhari and D. Patinkin, « The Role of Money in a Simple Growth Model », *American Economic Review*, Vol. 55 (September 1968), pp. 713-753.

([4]) Milton Friedman used this approach in his essay « The Optimum Quantity of Money » which appears as essay § 1 in M. Friedman, *The Optimum Quantity of Money and Other Essays*, Chicago, 1969.

([5]) For this comment see F. Hahn, « On Money and Growth », *Journal of Money, Credit and Banking*, Vol. 1 (May 1969), p. 172.

II. *Notation.*

The notation used in this model is the following: Variables:

C_{ij} = i^{th} output consumed by j^{th} person; $i = 1, 2, 3$, $j = 1, 2$
P_i = price of i^{th} output
P_4 = price of money viewed as a consumer good
P_K, P_L, P_M = price of capital, price of labor and price of money as a producer good, respectively
U_j = utility to the j^{th} person
X_i = i^{th} output
m_j = money considered as an output and demanded by the j^{th} person
K_i = capital absorbed by i^{th} output
L_i = labor absorbed by i^{th} output
M_i = money considered as an input and absorbed by i^{th} output
Y_j = income of the j^{th} person

Parameters:

A, B, C, D = elasticities of utility with respect to X_1, X_2, X_3 and real cash balances viewed as a consumer good, respectively
u_i = elasticity of i^{th} output with respect to capital
λ_i = elasticity of i^{th} output with respect to labor
μ_i = elasticity of i^{th} output with respect to real cash balances viewed as a producer good
a = proportion of total real cash balances held by person 1
K = total available quantity of capital
L = total available quantity of labor
M_T = total available supply of money

III. *The Model.*

The production functions of the model are of the Cobb-Douglas form with constant returns to scale. Real cash balances are introduced in the production function to express the function of money as a producer good. More specifically, the production functions of the three outputs are as follows:

$$(1) \quad X_1 = K_1^{u_1} \; L_1^{\lambda_1} \left(\frac{M_1}{P_L}\right)^{\mu_1}, \; X_1 > 0, \; K_1 > 0, \; L_1 > 0, \; \frac{M_1}{P_L} > 0, \; 0 < u_1, \lambda_1, \mu_1 < 1$$

$$(2) \quad X_2 = K_2^{u_2} \; L_2^{\lambda_2} \left(\frac{M_2}{P_L}\right)^{\mu_2}, \; X_2 > 0, \; K_2 > 0, \; L_2 > 0, \; \frac{M_2}{P_L} > 0, \; 0 < u_2, \lambda_2, \mu_2 < 1$$

$$(3) \quad X_3 = K_3^{u_3} \; L_3^{\lambda_3} \left(\frac{M_3}{P_L}\right)^{\mu_3}, \; X_3 > 0, \; K_3 > 0, \; L_3 > 0, \; \frac{M_3}{P_L} > 0, \; 0 < u_3, \lambda_3, \mu_3 < 1$$

Assume that firms seek to maximize profits under conditions of perfect competition. The first order conditions of an unconstrained perfect competitive maximization of profits are presented in the following equations:

$$P_K = P_1 \cdot \frac{u_1}{K_1} \cdot X_1 \tag{4}$$

$$P_L = P_1 \cdot \frac{\lambda_1}{L_1} \cdot X_1 \tag{5}$$

$$P_M = P_1 \cdot \frac{\mu_1}{\left(\frac{M_1}{P_L}\right)} \cdot X_1 \tag{6}$$

$$P_K = P_2 \cdot \frac{u_2}{K_2} \cdot X_2 \tag{7}$$

$$P_L = P_2 \cdot \frac{\lambda_2}{L_2} \cdot X_2 \tag{8}$$

$$P_M = P_2 \cdot \frac{\mu_2}{\left(\frac{M_2}{P_L}\right)} \cdot X_2 \tag{9}$$

$$P_K = P_3 \cdot \frac{u_3}{K_3} \cdot X_3 \tag{10}$$

$$P_L = P_3 \cdot \frac{\lambda_3}{L_3} \cdot X_3 \tag{11}$$

$$P_M = P_3 \cdot \frac{\mu_3}{\left(\frac{M_3}{P_L}\right)} \cdot X_3 \tag{12}$$

Euler's equations are obtained for convenience in deriving the algebraic solutions of the model in section IV.

$$P_1 X_1 = P_K K_1 + P_L L_1 + P_M \left(\frac{M_1}{P_L}\right) \tag{4a}$$

$$P_2 X_2 = P_K K_2 + P_L L_2 + P_M \left(\frac{M_2}{P_L}\right) \tag{7a}$$

$$P_3 X_3 = P_K K_3 + P_L L_3 + P_M \left(\frac{M_3}{P_L}\right) \tag{10a}$$

Having presented the production aspects of the model, emphasis is given next to the consumption side. It is assumed that there are two consumers having the same utility function which is of the Cobb-Douglas form. Money is introduced in the utility function to account for the hypothesis that real cash balances function as a consumer good. The utility functions with the budget constraints are presented below.

$$U_1 = C_{11}^A C_{21}^B C_{31}^C \left(\frac{m_1}{P_L}\right)^D$$

$$U_2 = C_{12}^A C_{22}^B C_{32}^C \left(\frac{m_2}{P_L}\right)^D$$

$$Y_1 = P_1 C_{11} + P_2 C_{21} + P_3 C_{31} + P_4 \left(\frac{m_1}{P_L}\right)$$

$$Y_2 = P_1 C_{12} + P_2 C_{22} + P_3 C_{32} + P_4 \left(\frac{m_2}{P_L}\right)$$

To obtain the demand function of the model, maximize utility under the above constraints.

$$C_{11} = \frac{Y_1}{P_1} \cdot \frac{A}{A+B+C+D} \tag{13}$$

$$C_{21} = \frac{Y_1}{P_2} \cdot \frac{B}{A+B+C+D} \tag{14}$$

$$C_{31} = \frac{Y_1}{P_3} \cdot \frac{C}{A+B+C+D} \tag{15}$$

$$\frac{m_1}{P_L} = \frac{Y_1}{P_4} \cdot \frac{D}{A+B+C+D} \tag{16}$$

$$C_{12} = \frac{Y_2}{P_1} \cdot \frac{A}{A+B+C+D} \tag{17}$$

$$C_{22} = \frac{Y_2}{P_2} \cdot \frac{B}{A+B+C+D} \tag{18}$$

$$C_{32} = \frac{Y_2}{P_3} \cdot \frac{C}{A+B+C+D} \tag{19}$$

$$\frac{m_2}{P_L} = \frac{Y_2}{P_4} \cdot \frac{D}{A+B+C+D} \tag{20}$$

For the model to be completed, a definition of personal incomes is necessary and the equilibrium conditions need to be stated. First consider the personal incomes.

$$Y_1 = P_K K + a P_M \left(\frac{M_T}{P_L}\right) \tag{21}$$

$$Y_2 = P_L L + (1 - a) P_M \left(\frac{M_T}{P_L}\right) \tag{22}$$

Equations (21) and (22) show that the first person is assumed to be the capitalist, while the second is assumed to be the worker. Note, however, that equations (21) and (22) contain an additional term. This term reveals that the initial stock of real cash balances has been distributed between the two persons according to some fixed proportion a.

The equilibrium conditions in the input markets are stated in equations (23) - (25).

$$K = K_1 + K_2 + K_3 \tag{23}$$

$$L = L_1 + L_2 + L_3 \tag{24}$$

$$\frac{M}{P_L} = \frac{M_1}{P_L} + \frac{M_2}{P_L} + \frac{M_3}{P_L} \tag{25}$$

Furthermore, equations (26) - (28) describe equilibrium conditions in output markets.

$$X_1 = C_{11} + C_{12} \tag{26}$$

$$X_2 = C_{21} + C_{22} \tag{27}$$

$$X_3 = C_{31} + C_{32} \tag{27a}$$

$$\frac{m}{P_L} = \frac{m_1}{P_L} + \frac{m_2}{P_L} \tag{28}$$

Equation (27a) simply indicates that equilibria in the output markets X_1 and X_2 would automatically force equilibrium on output market X_3. Thus equation (27a) does not add any necessary information for the solution of the model. It simply indicates that Walras' law holds true.

Finally, some emphasis is given to the money market. Equations (29) and (30) describe the equilibrium conditions in the money market. They indicate that total supply of real cash balances is equal to the demand of real cash balances by the production sector where money is viewed as an input, plus the demand of real cash balances demanded by the consumer sector, where money is viewed as a consumption good.

$$\frac{M_T}{P_L} = \frac{M}{P_L} + \frac{m}{P_L} \tag{29}$$

$$P_4 = P_M \tag{30}$$

Note that equation (30) implies that the price of real cash balances in a state of equilibrium is equated to the rate of return of real cash balances ([6]).

([6]) Note that the interest rate that producers pay for using money need not be the same

The model presented in equations (1) - (30) consists of 30 equations and 31 unknowns. The unknowns are: L_1, L_2, L_3, K_1, K_2, K_3, M_1, M_2, M_3, X_1, X_2, X_3, P_1, P_2, P_3, P_4, p_K, p_L, p_M, Y_1, Y_2, C_{11}, C_{12}, C_{22}, C_{21}, C_{31}, C_{32}, m_1, m_2, m, M. Let P_L be the Walrasian numeraire (7). In what follows, a set of solutions of the model is presented with an indication of the algebraic manipulations involved.

V. *Solutions of the Model.*

The solutions of L_1, L_2, L_3, K_1, K_2 and K_3 are obtained in a straightforward manner by making use of equations (4a), (6a), (8a), (13) - (20), (26) - (27a) as well as equations (4) - (12), (23) and (24). Observe that the allocation of capital as well as labor is not affected by the introduction of real cash balances in the production functions. The solutions are:

$$L_1 = \frac{A\lambda_1}{A\lambda_1 + B\lambda_2 + C\lambda_3} \cdot L \tag{31}$$

$$L_2 = \frac{B\lambda_2}{A\lambda_1 + B\lambda_2 + C\lambda_3} \cdot L \tag{32}$$

$$L_3 = \frac{C\lambda_3}{A\lambda_1 + B\lambda_2 + C\lambda_3} \cdot L \tag{33}$$

$$K_1 = \frac{Au_1}{Au_1 + Bu_2 + Cu_3} \cdot K \tag{34}$$

$$K_2 = \frac{Bu_2}{Au_1 + Bu_2 + Cu_3} \cdot K \tag{35}$$

$$K_3 = \frac{Cu_3}{Au_1 + Bu_2 + Cu_3} \cdot K \tag{36}$$

For the solutions of money as an input, some preliminary algebraic steps are necessary.

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as the interest rate that consumers pay for using money in cases where the economic system is off equilibrium. Equation (30) however forces the equality of the two interest rates as an equilibrium condition.

(7) The use of the price of labor, i.e., wage rate, as a numeraire for the model is consistent with both the Keynesian tradition and the derivation of meaningful results. For suppose that the wage rate was allowed to vary; then the units of the economy motivated by utility maximization considerations would attempt to increase the real quantities of economic goods by reducing the wage rate. This would imply that utility maximization would eventually drive the wage rate to zero; such an effect is obviously unobservable in economic reality.

Use Euler's equations (4a), (7a) and (10a) and perform the necessary algebraic computations to obtain:

$$BC\left(P_K K_1 + P_L L_1 + P_M \frac{M_1}{P_L}\right) = AC\left(P_K K_2 + P_L L_2 + P_M \frac{M_2}{P_L}\right)$$
$$= AB\left(P_K K_3 + P_L L_3 + P_M \frac{M_3}{P_L}\right)$$

Also from equations (4) - (12) solve to get the following results:

$$\frac{P_K}{P_M} = \frac{u_1}{\mu_1} \cdot \frac{\left(\frac{M_1}{P_L}\right)}{K_1} = \frac{u_2}{\mu_2} \cdot \frac{\left(\frac{M_2}{P_L}\right)}{K_2} = \frac{u_3}{\mu_3} \cdot \frac{\left(\frac{M_3}{P_L}\right)}{K_3}$$

$$\frac{P_L}{P_M} = \frac{\lambda_1}{\mu_1} \cdot \frac{\left(\frac{M_1}{P_L}\right)}{L_1} = \frac{\lambda_2}{\mu_2} \cdot \frac{\left(\frac{M_2}{P_L}\right)}{L_2} = \frac{\lambda_3}{\mu_3} \cdot \frac{\left(\frac{M_3}{P_L}\right)}{L_3}$$

Combine the above equations with equations (23), (24) and (25) to solve for real cash balances as inputs. The solutions obtained are:

$$(37) \qquad \frac{M_1}{P_L} = \frac{A\mu_1}{A\mu_1 + B\mu_2 + C\mu_3}\left(\frac{M}{P_L}\right)$$

$$(38) \qquad \frac{M_2}{P_L} = \frac{B\mu_2}{A\mu_1 + B\mu_2 + C\mu_3}\left(\frac{M}{P_L}\right)$$

$$(39) \qquad \frac{M_3}{P_L} = \frac{C\mu_3}{A\mu_1 + B\mu_2 + C\mu_3}\left(\frac{M}{P_L}\right)$$

Direct insertion of input solutions presented in equations (31) - (39) into the production functions of the model yields the following output solutions:

$$(40) \qquad X_1 = \left[\frac{AKu_1}{Au_1 + Bu_2 + Cu_3}\right]^{u_1}\left[\frac{AL\lambda_1}{A\lambda_1 + B\lambda_2 + C\lambda_3}\right]^{\lambda_1}\left[\frac{A\left(\frac{M}{P_L}\right)\mu_1}{A\mu_1 + B\mu_2 + C\mu_3}\right]^{\mu_1}$$

$$(41) \qquad X_2 = \left[\frac{BKu_2}{Au_1 + Bu_2 + Cu_3}\right]^{u_2}\left[\frac{BL\lambda_2}{A\lambda_1 + B\lambda_2 + C\lambda_3}\right]^{\lambda_2}\left[\frac{B\left(\frac{M}{P_L}\right)\mu_2,}{A\mu_1 + B\mu_2 + C\mu_3}\right]^{\mu_2}$$

(42)
$$X_3 = \left[\frac{CKu_3}{Au_1 + Bu_2 + Cu_3}\right]^{u_3} \left[\frac{CL\lambda_3}{A\lambda_1 + B\lambda_2 + C\lambda_3}\right]^{\lambda_3} \left[\frac{C\left(\frac{M}{P_L}\right)\mu_3}{A\mu_1 + B\mu_2 + C\mu_3}\right]^{\mu_3}$$

The solutions for relative prices are obtained with the help of equations (4) - (12) and the input and output solutions presented above. In what follows, the input and output solutions have not been inserted into the relative prices solutions to eliminate unduly lengthy expressions.

(43)
$$\frac{P_1}{P_L} = \frac{L_1}{\lambda_1} \cdot \frac{1}{X_1}$$

(44)
$$\frac{P_2}{P_L} = \frac{L_2}{\lambda_2} \cdot \frac{1}{X_2}$$

(45)
$$\frac{P_3}{P_L} = \frac{L_3}{\lambda_3} \cdot \frac{1}{X_3}$$

(46)
$$\frac{P_4}{P_L} = \frac{P_M}{P_L} \cdot \frac{\mu_1}{\lambda_1} \cdot \frac{L_1}{\left(\frac{M_1}{P_L}\right)}$$

(47)
$$\frac{P_K}{P_L} = \frac{u_1}{\lambda_1} \cdot \frac{L_1}{K_1}$$

The solutions of real incomes are in order. Use equations (21) and (22) that define absolute incomes. Divide equations (21) and (22) by p_L and use the solutions for relative prices presented above. Note again that direct insertions of solutions already discussed above have not been made for the sake of simplicity.

(48)
$$\frac{Y_1}{P_L} = \frac{P_K}{P_L} \cdot K + a \quad \frac{P_M}{P_L} \cdot \left(\frac{M_T}{P_L}\right)$$

(49)
$$\frac{Y_2}{P_L} = \frac{P_L}{P_L} \cdot L + (1 - a)\frac{P_M}{P_L} \cdot \left(\frac{M_T}{P_L}\right)$$

In order to obtain the solutions for consumption, use equations (13) - (20) and several of the solutions already presented above. As before, note that not all necessary insertions are made.

(50)
$$C_{11} = \frac{Au_1 + Bu_2 + Cu_3}{A + B + C + D} \cdot X_1 + a \cdot \frac{A\mu_1 + B\mu_2 + C\mu_3}{A + B + C + D} \cdot X_1$$

(51)
$$C_{21} = \frac{Au_1 + Bu_2 + Cu_3}{A + B + C + D} \cdot X_2 + a \cdot \frac{A\mu_1 + B\mu_2 + C\mu_3}{A + B + C + D} \cdot X_2$$

$$(52)\qquad C_{31} = \frac{Au_1 + Bu_2 + Cu_3}{A + B + C + D} \cdot X_3 + a \cdot \frac{A\mu_1 + B\mu_2 + C\mu_3}{A + B + C + D} \cdot X_3$$

$$(53)\qquad \frac{m_1}{P_L} = \frac{D}{A + B + C + D}\left[\frac{\mu_1 (Au_1 + Bu_2 + Cu_3)}{u_1 (A\mu_1 + B\mu_2 + C\mu_3)} \cdot \frac{M}{P_L} + a\frac{M_T}{P_L}\right]$$

$$(54)\qquad C_{12} = \frac{A\lambda_1 + B\lambda_2 + C\lambda_3}{A + B + C + D} \cdot X_1 + (1 - a) \cdot \frac{A\mu_1 + B\mu_2 + C\mu_3}{A + B + C + D} \cdot X_1$$

$$(55)\qquad C_{22} = \frac{A\lambda_1 + B\lambda_2 + C\lambda_3}{A + B + C + D} \cdot X_2 + (1 - a) \cdot \frac{A\mu_1 + B\mu_2 + C\mu_3}{A + B + C + D} \cdot X_2$$

$$(56)\qquad C_{32} = \frac{A\lambda_1 + B\lambda_2 + C\lambda_3}{A + B + C + D} \cdot X_3 + (1 - a) \cdot \frac{A\mu_1 + B\mu_2 + C\mu_3}{A + B + C + D} \cdot X_3$$

$$(57)\qquad \frac{m_2}{P_L} = \frac{D}{A+B+C+D}\left[\frac{\mu_1 (A\lambda_1 + B\lambda_2 + C\lambda_3)}{\lambda_1 (A\mu_1 + B\mu_2 + C\mu_3)} \cdot \frac{M}{P_L} + (1-a)\frac{M_T}{P_L}\right]$$

The last two solutions below refer to the distribution of real cash balances between their uses as consumer good and producer good. To obtain such solutions, use equations (53), (57) and (29).

$$(58)\qquad \frac{M}{P_L} = \frac{(A+B+C+D)\lambda_1 u_1 (A\mu_1 + B\mu_2 + C\mu_3) \cdot \left(\frac{M_T}{P_L}\right)}{(A + B + C + D)\lambda_1 u_1 (A\mu_1 + B\mu_2 + C\mu_3) + \lambda_1\mu_1 D (Au_1 + Bu_2 + Cu_3) + u_1\mu_1 D (A\lambda_1 + B\lambda_2 + C\lambda_3)}$$

Finally, use equations (29) and (58) to get:

$$(59)\qquad \frac{m}{P_L} = \frac{M_T}{P_L} - \frac{M}{P_L}$$

Thus the model is presented and a set of solutions is obtained. The remainder of this paper is devoted to some interpreting comments.

VI. *Interpreting comments.*

With the help of a simple, solvable, static general equilibrium model of the Cobb-Douglas form it has been demonstrated that real cash balances could be conveniently accomodated by being introduced as mathematical arguments in the utility and production functions of the model. Such an introduction provides a theoretical justification to the development of a monetary economy and allows for comparison between barter economies and monetary ones. It is evident that total output and total utility are higher in a monetary economy and that is due directly to the fact that money contributes to output increases by being viewed as a consumer good.

Under the assumption of profit maximization, the present model yields not only input demand functions for capital and labor but also demand functions for money viewed as an input. Furthermore the assumption of constrained utility maximization yields demand functions not only for output but also for real cash balances viewed as a consumer good.

The significance of the introduction of money is witnessed by the way money and monetary constants, i.e., output elasticities with respect to real cash balances viewed as a producer good and utility elasticities with respect to real cash balances viewed as a consumer good, are present in most equations of the model. More specifically, the utility elasticity with respect to real cash balances viewed as a consumer good affects all the demand functions and consequently the worker's and the capitalist's consumption of the three consumption goods.

Finally, the present model possesses a rather simple but important monetary market where the total supply of money interacts with the two sources of demand for money, i.e., demand for money viewed as a consumer good and demand for money viewed as a producer good, to determine an equilibrium price for the services of money. Finally, the allocation of real cash balances between their two uses is determined by all the utility and all the output elasticities which corresponds to what one might have anticipated.

UN MODELLO CON DISPONIBILITA' MONETARIE REALI NELLA FUNZIONE DI PRODUZIONE E DI UTILITA'

Scopo di questo saggio è di sviluppare, risolvere e discutere un modello di equilibrio generale statico basato su termini reali. Si spera che esso fornisca qualche spunto utile allo sforzo dell'economista di integrare la teoria monetaria con quella del valore.

La paternità intellettuale di questo studio risale al lavoro di Patinkin, *Money, Interest and Prices*. In esso e altrove Patinkin ha sostenuto che è teoricamente giustificato considerare le disponibilità reali sia come bene di consumo che danno utilità al loro detentore che come beni di produzione, che forniscono servizi produttivi al produttore. Questo approccio è stato adottato da diversi economisti monetari. E' vero tuttavia che certe difficoltà concettuali permangono.

Si consideri un'economia di baratto con tre beni di consumo e due fattori di produzione (*inputs*), capitale e lavoro. Questa economia soffra di inefficienze economiche sia nella produzione che nello scambio dei beni. Sebbene essa non sia complessa come le economie moderne, la « doppia coincidenza » dei bisogni non si afferma in modo molto naturale. L'introduzione della moneta facilita lo scambio e contribuisce a una produzione piú efficiente. Il teorico economico solleverebbe immediatamente la questione: Come possiamo spiegare teoricamente l'introduzione della moneta in un'economia di baratto e come possiamo provare che la moneta influenza le soluzioni di un modello di baratto?

Nel corpo dell'articolo viene dimostrato che se la moneta è considerata tanto come bene di consumo quanto come bene di produzione, allora essa può essere introdotta nella forma di disponibilità reali come argomento delle funzioni di produzione e di utilità di un modello matematico e i suoi effetti possono essere individuati con lo sviluppo delle soluzioni del modello. Poiché siamo interessati alla dimostrazione, anziché scegliere una prova astratta seguiamo un modello di equilibrio generale risolvibile come quello di Cobb-Douglas. Per ragioni di semplicità viene seguita un'analisi statica anziché dinamica o addirittura stocastica. Le notazioni del modello sono date nella sezione II. Il modello è descritto nella sezione III. Le sezioni IV e V sono dedicate rispettivamente alle soluzioni del modello e ad alcune osservazioni interpretative.

Nell'ipotesi della massimizzazione del profitto questo modello non dà soltanto funzioni di domanda d'investimento di capitale e lavoro, ma anche funzioni di domanda di moneta considerata come input, cioè come fattore di produzione. Inoltre l'ipotesi della massimizzazione vincolata dell'utilità dà la funzione di domanda non solo della produzione ma anche delle disponibilità monetarie reali considerate come beni di consumo.

L'importanza dell'introduzione della moneta è testimoniata dal fatto che moneta e costanti monetarie sono presenti nella maggior parte delle equazioni del modello. Le elasticità di produzione rispetto alle disponibilità monetarie reali sono cioè considerate come beni di produzione e le elasticità dell'utilità rispetto alle disponibilità monetarie reali sono considerate come beni di consumo. Piú specificamente, l'elasticità dell'utilità rispetto alle disponibilità monetarie reali considerate come beni di consumo influenza tutte le funzioni di domanda e conseguentemente il consumo dei tre beni di consumo da parte dei lavoratori e del capitalista.

Infine questo modello possiede un mercato monetario piuttosto semplice ma importante, dove l'offerta totale di moneta interagisce con le due fonti di domanda monetaria, cioè domanda di moneta considerata come bene di consumo e domanda di moneta considerata come bene di produzione, per determinare il prezzo di equilibrio dei servizi monetari. Infine, l'allocazione delle disponibilità monetarie reali tra i loro due usi è determinata da tutte le elasticità dell'utilità e della produzione, ciò che corrisponde a una condizione intuitiva.