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FORWARD AND FUTURES MARKETS

Conceptual differences and computational illustrations

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This chapter develops the essential elements of the conceptual differences between forward and futures prices and offers several computational illustrations. The conceptual differences are expressed precisely in mathematical equations which specially demonstrate the similarities and differences in these two concepts. To study the impact of the independent variables that enter the equations for futures and forward pricing of contracts, several computational examples are presented, which could not be performed easily without computers.

INTRODUCTION

The financial uncertainty of the late 1970s and the 1980s has generated great interest in the futures and forward markets. Both markets can be used by a firm which owns an asset and wishes to protect its value from potential price decline. The firm could hedge the value of its asset by selling a forward or futures contract at the current price of its asset with an agreement to deliver such an asset at a prespecified future date. If, between now and the delivery date, the price of the asset declines, the firm is perfectly protected because of its forward or futures contract which has locked in the current price. However, if the price of the asset increases, by analogy, the firm could not enjoy any windfall profit because the futures or forward contract has prespecified the price.

To the nonspecialist, forward and futures markets are similar. Actually, numerous empirical studies have compared futures and forward prices and have often been unable to confirm always statistically significant differences. For example, Puglisi (1978), Capozza and Cornell (1979), Rendleman and Carabini (1979) and Vignola and Dale (1980) compare the prices for treasury bill futures contracts with the forward prices implied by the interest rates of treasury bills traded in the spot market. These studies do

not detect specific and consistent differences between futures and forward prices for the US treasury bill market. Similarly, Cornell and Reinganum (1981) compare futures and forward prices for foreign exchange markets. They find that the foreign data reveal that mean differences between forward and futures prices are insignificantly different from zero in a statistical sense. The mean discrepancy found is less than the mean bid–ask spread in the forward market in 18 of the 20 cases studied, and it barely exceeds it in the remaining two cases. On the other hand, French (1983) claims that there are significant differences between futures and forward prices in the copper and silver markets.

It is the purpose of this chapter to use decision-making and simulation techniques to illustrate numerical differences and similarities between forward and futures prices. Cox *et al.* (1981), Jarrow and Oldfield (1981), French (1983) and others develop detailed theoretical models of forward and futures prices but do not do any simulations. Thus, this chapter demonstrates the positive impact of computing in teaching business students the subtle differences between the two important concepts of forward and futures contracts. Although standard textbooks, such as Kolb (1994), introduce these concepts they do so without any mathematical derivations or any computational illustrations.

The next section develops the analysis of a forward price while the third section presents futures pricing. Using Lotus 1–2–3, several numerical examples simulate forward and futures pricing in the fourth section. Conclusions are presented in the final section.

FORWARD PRICES

A firm which purchases a forward contract agrees to buy a specific asset on a specified future date for an agreed price. This price is called the forward price.

To motivate the pricing of a forward contract consider the simple two-period case. Let t denote today and $t + 1$, $t + 2$ denote the subsequent two periods. The contract will be executed on period $t + 2$. Denote by V_{ij} the price of the asset at period $t + 2$ when the economy is at state i at $t + 1$ and at state j at $t + 2$. Assume that both i = 1,2 and j = 1,2. In other words, we assume that the economy experiences two states in each period. Allowing more states is straightforward.

There are two relevant economic variables: V_{ij}, i.e., the price of the asset, and r_{ij}, i.e., interest rates. Figure 13.1 illustrates that moving from period t, now, to period $t + 1$, tomorrow, the economy's interest rates will go either to r_1, with probability P_1 or to r_2 with probability $P_2 = 1 - P_1$. From period $t + 1$ to $t + 2$, interest rates will go to r_{11} or r_{12}, provided they were at r_1 at $t + 1$ (with probabilities P_{11} and P_{12} respectively) or to r_{21} or r_{22} (with probabilities P_{21} and P_{22} respectively) provided they were at r_2 at $t + 1$.

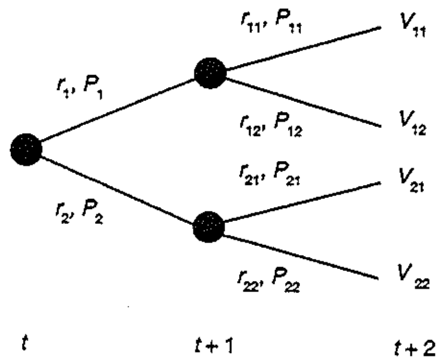

Figure 13.1 Two-state probability tree (1)

The V_{ij} denote expectations about the price of the asset at time j provided that state i of the economy materialized at time $t + 1$.

Since no money changes hands at time t, both the buyer and the seller of the forward contract are willing to transact if the present values of the expected price V_{ij} and the forward price, denoted $G(t)$ are equal. Note that although $G(t)$, the forward price, is agreed upon today (at time t) it is paid at time $t + 2$; this explains why we consider present values. Such present values are given by

$$G(t)\left[\frac{P_1P_{11}}{(1+r_1)(1+r_{11})}+\frac{P_1P_{12}}{(1+r_1)(1+r_{12})}+\frac{P_2P_{21}}{(1+r_2)(1+r_{21})}+\frac{P_2P_{22}}{(1+r_2)(1+r_{22})}\right]$$

$$=\frac{V_{11}P_1P_{11}}{(1+r_1)(1+r_{11})}+\frac{V_{12}P_1P_{12}}{(1+r_1)(1+r_{12})}+\frac{V_{21}P_2P_{21}}{(1+r_2)(1+r_{21})}+\frac{V_{22}P_2P_{22}}{(1+r_2)(1+r_{22})} \quad (1)$$

From (1) we can immediately solve the forward price $G(t)$ given below

$$G(t)=\frac{\dfrac{V_{11}P_1P_{11}}{(1+r_1)(1+r_{11})}+\dfrac{V_{12}P_1P_{12}}{(1+r_1)(1+r_{12})}+\dfrac{V_{21}P_2P_{21}}{(1+r_2)(1+r_{21})}+\dfrac{V_{22}P_2P_{22}}{(1+r_2)(1+r_{22})}}{\dfrac{P_1P_{11}}{(1+r_1)(1+r_{11})}+\dfrac{P_1P_{12}}{(1+r_1)(1+r_{12})}+\dfrac{P_2P_{21}}{(1+r_2)(1+r_{21})}+\dfrac{P_2P_{22}}{(1+r_2)(1+r_{22})}} \quad (2)$$

Having obtained the forward price for the simplified two-state, two-period case we next present the futures price.

FUTURES PRICE

Let $H(t)$ denote the futures price agreed upon today to be executed at period $t + 2$. What makes the computation of the futures price interesting is the institutional procedure called *daily settlement*. According to this procedure,

administered by the Clearing Authorities of Futures Exchanges, the futures contract, both for the buyer and the seller is priced to market daily. Thus, if the futures price at $t + 1$ is H_i with $i = 1,2$ denoting the state of the economy, then unless $H_i = H(t)$, the party in whose favor the price moved by $H_i - H(t)$ must immediately be paid this amount by the losing party. Recall that such settlement does not occur in a forward market. The economic justification of daily settlement is explained by the desire of organized exchanges to reduce risk by allocating potential price changes across the life of the futures contract in lieu of a one-time settlement at the maturity of the contract.

Using the same decision theoretic diagram (Figure 13.1), note that daily settlement means that the H_i, $i = 1,2$ must be adjusted by the amount

$$H_i - H(t) \quad (3)$$

which, if appropriately discounted, should be a fair game with zero present value. In symbols,

$$\sum_i \frac{[H_i - H(t)]P_i}{1+r_i} = 0 \quad (4)$$

which yields that

$$H(t) = \frac{\dfrac{H_1P_1}{(1+r_1)}+\dfrac{H_2P_2}{(1+r_2)}}{\dfrac{P_1}{(1+r_1)}+\dfrac{P_2}{(1+r_2)}} \quad (5)$$

So far, it appears that $H(t)$ in (5) resembles (2). However, note that H_1 and H_2 in (5) are each discounted values of the asset prices expected to prevail at $t + 2$. For $i = 1,2$, observe that

$$H_i = \frac{\dfrac{V_{i1}P_{i1}}{1+r_{i1}}+\dfrac{V_{i2}P_{i2}}{1+r_{i2}}}{\dfrac{P_{i1}}{1+r_{i1}}+\dfrac{P_{i2}}{1+r_{i2}}} \quad (6)$$

Put H_i, $i = 1,2$ of (6) in (5) and perform the necessary algebra to conclude that

$$G(t)=\frac{\dfrac{V_{11}P_1P_{11}}{(1+r_1)(1+r_{11})}+\dfrac{V_{12}P_1P_{12}}{(1+r_1)(1+r_{12})}+\dfrac{V_{21}P_2P_{21}}{(1+r_2)(1+r_{21})}+\dfrac{V_{22}P_2P_{22}}{(1+r_2)(1+r_{22})}}{\left[\dfrac{P_1}{1+r_1}+\dfrac{P_2}{1+r_2}\right]\left[\dfrac{P_1P_{11}}{1+r_{11}}+\dfrac{P_1P_{12}}{1+r_{12}}+\dfrac{P_2P_{21}}{1+r_{21}}+\dfrac{P_2P_{22}}{1+r_{22}}\right]} \quad (7)$$

A simple comparison of (2) and (7) shows that both expressions have the same numerator. Therefore differences or similarities between $G(t)$ and $H(t)$ depend on the denominator. Fisher Black (1976) showed that when interest rates are nonstochastic, i.e. constant, then $G(t) = H(t)$. This is trivial to see from the explicit expressions in (2) and (7). In general, however, forward prices, $G(t)$, need not be equal to futures prices, $H(t)$.

SIMULATIONS

Even in the simple two-period, two-state economy with varying expected prices at $t+2$ and varying interest rates and probabilities, it is cumbersome to perform calculations according to (2) and (7). Such calculations however, are obtained easily using Lotus 1–2–3.

Example 1

As a first simulation example, consider Figure 13.2. This example illustrates that short-term interest rates have equal probability 1/2 to drop to 5 percent or rise to 9 percent. If the former happens, then there is probability 2/3 that the rate will only increase to 6 percent and a smaller probability 1/3 that it will increase to 8 percent. If the latter occurs, the rates will decrease to 8 percent with probability 1/3 and will decrease even further to 7 percent with higher probability 2/3. The expected values of the asset, depending on what is anticipated during $t+1$, are given as 420, 200, 150, and 75. The Lotus 1–2–3 results are:

$$G(t) = \frac{125.7861 + 29.3948 + 21.2368 + 21.4353}{0.2994901 + 0.146972 + 0.141578 + 0.285804} = 226.4168$$

$$H(t) = \frac{125.7861 + 29.3948 + 21.2368 + 21.4353}{0.934906 + 0.934633} = 226.4293$$

with an insignificant difference of 0.006 percent.

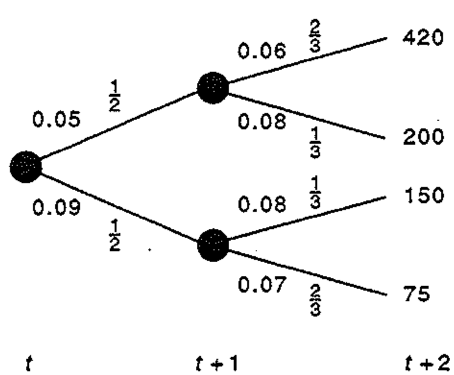

Figure 13.2 Two-state probability tree (2)

Example 2

Figure 13.3 illustrates the effects of changing interest rates. Note that the various probabilities and terminal values of the asset remain the same as in Example 1.

The Lotus 1–2–3 results are:

$$G(t) = \frac{107.0336 + 26.5922 + 23.7959 + 24.0315}{0.254841 + 0.132961 + 0.158639 + 0.320420} = 209.3221$$

$$H(t) = \frac{107.0336 + 26.5922 + 23.7959 + 24.0315}{0.944152 + 0.916137} = 209.7791$$

Note that as a result of changing interest rates, the difference between forward and futures prices has increased by 0.218 percent.

Example 3

In contrast to Examples 1 and 2, we now study again the influence of interest rates by allowing such rates to vary significantly. Specifically consider Figure 13.4.

The Lotus 1–2–3 results are:

$$G(t) = \frac{83.3333 + 21.36752 + 22.89377 + 23.57378}{0.198412 + 0.106837 + 0.152625 + 0.314317} = 195.7655$$

$$H(t) = \frac{83.3333 + 21.36752 + 22.89377 + 23.57378}{0.892857 + 0.856589} = 197.6542$$

with a difference of 0.965 percent.

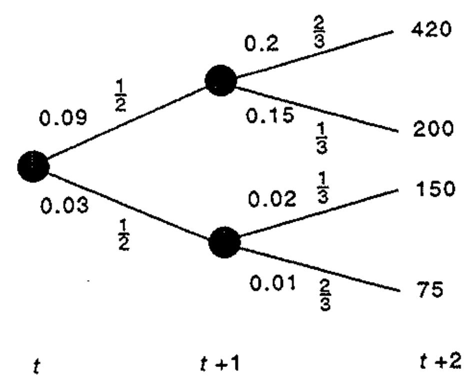

Figure 13.3 Two-state probability tree (3)

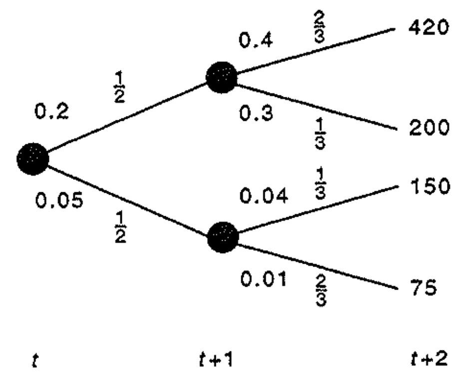

Figure 13.4 Two-state probability tree (4)

It is worth noting that two important results occur as interest rates change. First, the values of futures and forward prices change from around 226 in the first example to around 209 in the second example, and finally to around 195 in the third case. Second, the values of the forward and futures prices deviate because of the magnitude of interest rate changes.

Example 4

The complexity of the derived formulas for $H(t)$ and $G(t)$ does not allow straightforward analysis of the impact of changes in the inputs of the formula to the changes in the dependent forward and futures values. In this example we illustrate the impact on $H(t)$ and $G(t)$ of changes in the terminal values of V_{ij}, i=1,2, j=1,2. The Lotus 1–2–3 results are:

$$G(t) = \frac{83.3333 + 43.80341 + 61.05006 + 122.5836}{0.198412 + 0.106837 + 0.152625 + 0.314317} = 402.4397$$

$$H(t) = \frac{83.3333 + 43.80341 + 61.05006 + 122.5836}{0.892857 + 0.856589} = 406.3356$$

with a difference of 0.965 percent. Observe that the range of expected spot prices V_{ij} influences forward and futures prices. In Example 3 the V_{ij} prices range from 75 to 420, while in Example 4 they range from 390 to 420 (Figure 13.5). The corresponding forward prices are 195.77 and 402.39. In accordance with our intuition, this simulation illustrates that the expected terminal values V_{ij} are a significant input in forward and futures asset pricing.

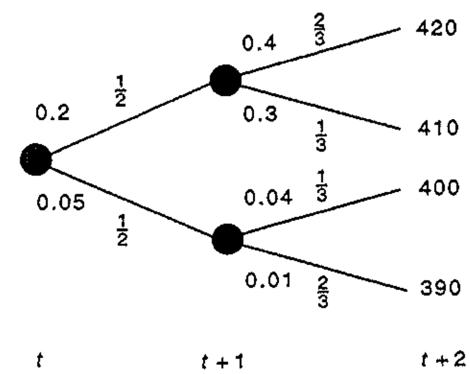

Figure 13.5 Two-state probability tree (5)

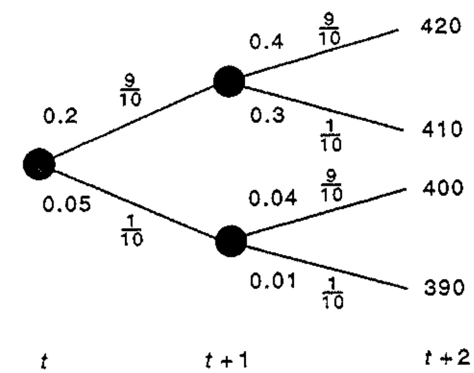

Figure 13.6 Two-state probability tree (6)

Example 5

The role of probability changes on the forward and futures prices is illustrated in this example. Consider Figure 13.6. The Lotus 1–2–3 results are:

$$G(t) = \frac{202.5000 + 23.65384 + 32.96703 + 3.677510}{0.482142 + 0.057692 + 0.082417 + 0.009429} = 416.0309$$

$$H(t) = \frac{202.5000 + 23.65384 + 32.96703 + 3.677510}{0.845238 + 0.744241} = 417.7627$$

with a difference of 0.417 percent. This example clarifies the role of probability distributions. More specifically, higher probabilities for higher terminal values led to higher forward and futures prices as contrasted between Examples 4 and 5.

CONCLUSIONS

This chapter presents the essential elements of the conceptual differences between forward and futures prices as developed in the financial literature. These conceptual differences are expressed precisely in mathematical equations which specifically show the similarities and differences in the two concepts. To study the impact of the independent variables that enter the equations for futures and forward pricing of contracts, several Lotus 1–2–3 simulations are presented to demonstrate the importance of computing in business education. The main conclusion is that currently the topic of forward and futures pricing can be studied rigorously by mathematical analysis and computing techniques. Although we used a two-period model, its generalization to n periods is straightforward.

REFERENCES

Black, F. (1976) "The Pricing of Commodity Contracts," *Journal of Financial Economics*, 3, 167–79.

Capozza, D. and B. Cornell (1979) "Treasury Bill Pricing in the Spot and Futures Markets," *Review of Economics and Statistics*, 61, 513–20.

Cornell, B. and M. Reinganum (1981) "Forward and Futures Prices: Evidence From the Foreign Exchange Markets," *Journal of Finance*, 36, 1035–45.

Cox, J., J. Ingersoll and S. Ross (1981) "The Relation Between Forward Prices and Futures Prices," *Journal of Financial Economics*, 9, 321–46.

French, K. (1983) "A Comparison of Futures and Forward Prices," *Journal of Financial Economics*, 12, 311–42.

Jarrow, R. and G. Oldfield (1981) "Forward Contracts and Futures Contracts," *Journal of Financial Economics*, 9, 373–82.

Kolb, R. (1994) *Understanding Futures Markets*, Fourth Edition, Miami, Fla.: Kolb Publishing Company.

Puglisi, D. (1978) "Is the Futures Market for Treasury Bills Efficient?," *Journal of Portfolio Management*, 4, Winter, 64–7.

Rendleman, R. and C. Carabini (1979) "The Efficiency of the Treasury Bill Futures Market," *Journal of Finance*, 39, 895–914.

Vignola, A. and C. Dale (1980) "The Efficiency of the Treasury Bill Futures Market: An Analysis of Alternative Specifications," *Journal of Financial Research*, 3, 169–88.