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Decomposition of Inflation and its Volatility: A Stochastic Approach

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Abstract. This paper presents a decomposition of inflation and its volatility. According to the traditional quantity theory of money, the rate of inflation is decomposed into three components: the rate of change in the money supply, plus the rate of change in the velocity of circulation, minus the rate of change in real output. We derive a generalization of this decomposition by postulating that the rate of change of money supply, velocity, and output follow diffusion equations. Using stochastic calculus techniques, two expressions are obtained decomposing inflation and its volatility as a sum of several economically important terms. We also use two sets of U.S. data to illustrate these decompositions with actual numbers.

Key words: inflation, volatility, quantity theory variables, stochastic modeling

1. Introduction

Inflation has played a significant role in economic history for as long as money has been used as a means of exchange. The importance of inflation as a topic of economic research is evidenced by the existing large literature surveyed earlier by Bronfenbrenner and Holzman (1963) and more recently by Parkin (1987).

This is a paper on inflation using a quantity theory of money framework. Friedman and Schwartz (1982) state that the quantity theory of money is "a theory that has taken many different forms and traces back to the very beginning of systematic thinking about economic matters." We do not wish here to review this theory in detail. Recently, Friedman (1987) has surveyed this theory very skillfully. In section 2, the quantity theory variables are introduced and the deterministic decomposition of inflation is obtained in equation (3). The extension of the quantity theory of money in the continuous time stochastic case is presented in section 3, and section 4 derives the stochastic decomposition of inflation and its volatility using the celebrated Itô's lemma. Section 5 offers a discussion of the generalization, and section 6 presents two numerical illustrations from two different data sets. The last section summarizes the paper and offers a conclusion about the contribution of this research.

2. Nonstochastic quantity theory of money

Let M, V, P, and Y denote the money stock, velocity of circulation, the implicit price deflator, and real gross national product respectively. The identity

$$MV \equiv PY \quad \text{or} \quad P \equiv MV/Y \tag{1}$$

has received great attention by economists. Fisher (1963) notes that: "This theory, though often crudely formulated, has been accepted by Locke, Hume, Adam Smith, Ricardo, Mill, Walker, Marshall, Fetter, Kemmerer, and most writers on the subject. The Roman, Julius Paulus, about 200 A.D., states his belief that the value of money depends upon its quantity."

We observe that economists have considered the variables M, V, P, and Y to be functions of time and by taking time derivatives they have obtained from (1)

$$\dot{P} = \frac{(\dot{M}V + \dot{V}M)Y - \dot{Y}MV}{Y^2} = \frac{\dot{M}V}{Y} + \frac{\dot{V}M}{Y} - \frac{\dot{Y}}{Y}\frac{MV}{Y} \tag{2}$$

where a dot above a variable denotes time derivative. From (2) we conclude that

$$\frac{\dot{P}}{P} = \frac{\dot{M}}{M} + \frac{\dot{V}}{V} - \frac{\dot{Y}}{Y} \tag{3}$$

which simply tells us that the proportional change in the price level is equal to the sum of the proportional change in the money stock, plus the proportional change in the velocity of money, minus the proportional change in real output. Note that in decomposing the rate of inflation into three terms, equation (3) relates sample averages during the estimation period.

3. Stochastic quantity theory of money

A casual observation of actual data for $\dot{M}/M$, $\dot{V}/V$, and $\dot{Y}/Y$ shows that these are not smooth deterministic variables. For example, figure 1 plots annual rates of change for money growth, real output, and velocity for 1947–1988, and suggests that these rates follow Itô processes. Following Merton (1975, 1982), we postulate that the behavior of the proportional rate

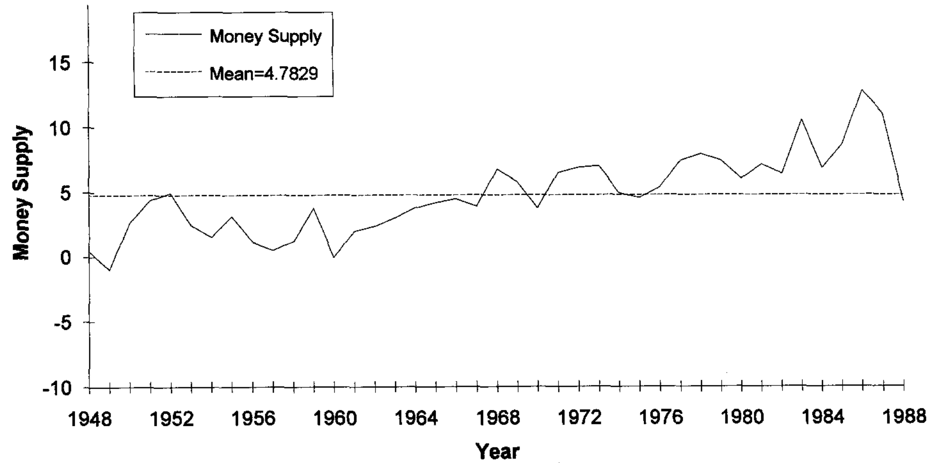

Figure 1. Annual rates of change for money growth, real output, and velocity for 1947–1988.

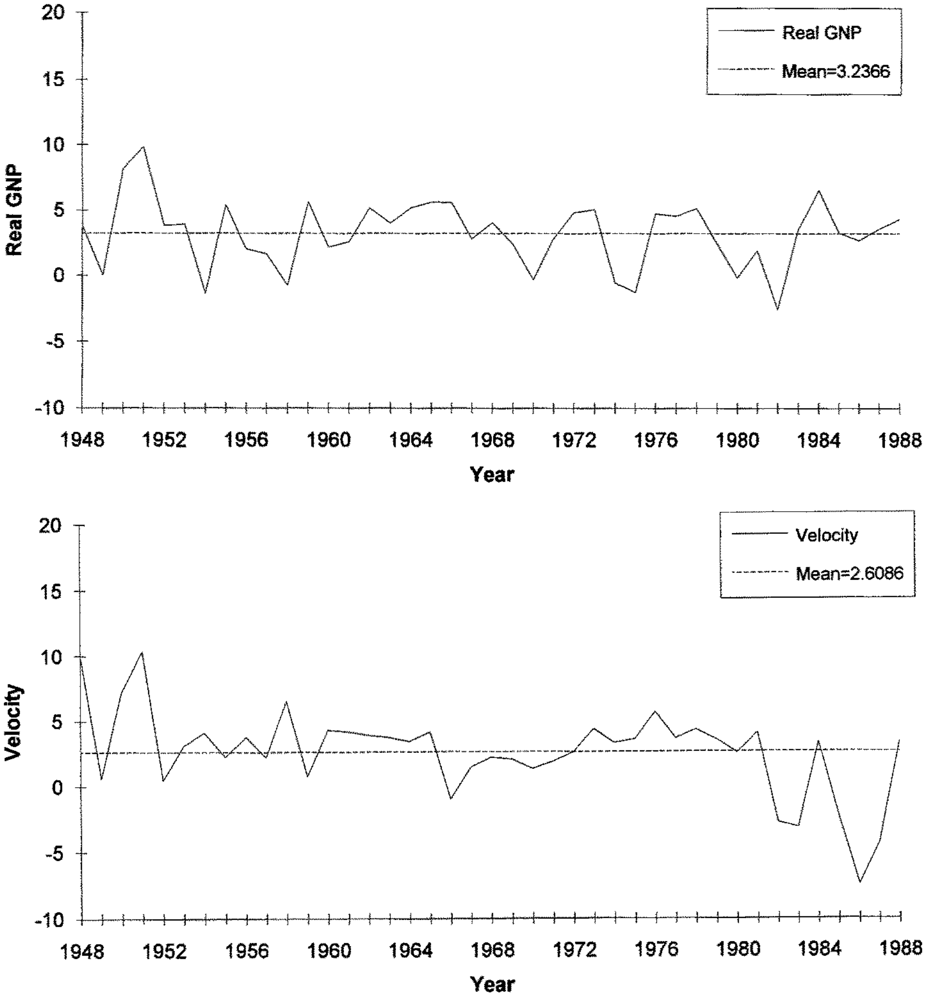

Figure 1. Continued.

of growth of money, velocity, and output are described by the Itô equations given in (4), (5), and (6).

$$\frac{dM(t)}{M(t)} = \mu_M(t, M)dt + \sigma_M(t, M)dZ_M(t) \tag{4}$$

$$\frac{dV(t)}{V(t)} = \mu_V(t, V)dt + \sigma_V(t, V)dZ_V(t) \tag{5}$$

$$\frac{dY(t)}{Y(t)} = \mu_Y(t, Y)dt + \sigma_Y(t, Y)dZ_Y(t). \tag{6}$$

Financial economists are familiar with Itô's equations from the theory of trading in continuous time, and macroeconomists, such as Fischer (1975), Gertler and Grinols (1982), or Grinols and Turnovsky (1993) have used such equations extensively. To save space, we do not review the elements of continuous time stochastic modeling. An extensive discussion, with numerous examples from economics and finance, can be found in Malliaris and Brock (1982). Gikhman and Skorokhod (1969) exposit the mathematical foundations of stochastic modeling in detail, while Malliaris (1983, 1990) and Merton (1982) give general overviews.

4. Decomposition of inflation and its volatility

Suppose that money supply, velocity of circulation, and real GNP follow stochastic processes described by the Itô equations (4)–(6). Heuristically, one may argue that the rate of inflation must also follow a process described by an Itô equation. This is true because of the generalized Itô's lemma which is stated below.

Generalized Itô's Lemma. Let n one-dimensional Itô processes $X_i(t)$ be given by

$$dX_i(t) = f_i(t)dt + \sigma_i(t)dZ_i(t), \qquad i = 1, 2, \ldots, n$$

Suppose that $u = u(t, x_1, x_2, \ldots, x_n)$: $[0, T] \times R^n \to R$ has partial derivatives

$$u_t, \quad u_{x_i}, \quad u_{x_i x_j}, \quad i, j \leq n$$

which are continuous. Then, the process

$$Y(t) = u[t, X_1(t), \ldots, X_n(t)]$$

is also an Itô process given by

$$dY(t) = u_t dt + \sum_{i=1}^{n} u_{x_i} dX_i + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} u_{x_i x_j} dX_i dX_j$$

where the product $dX_i dX_j$ can be calculated using the following multiplication rules:

$$dZ_i dZ_j = \rho_{ij} dt \qquad \text{for } i \neq j, \quad i, j \leq n;$$

$$dZ_i dZ_j = dt \qquad \text{for } i = j, \quad i, j, \leq n;$$

$$dt\,dZ_i = 0, \qquad \text{for } i = 1, 2, \ldots, n.$$

$$dt\,dt = 0.$$

Using Itô's lemma, equations (4)–(6) and the identity (1), several calculations yield that inflation follows at Itô process given by

$$\frac{dP}{P} = [\mu_M + \mu_V - \mu_Y + \sigma_M\sigma_V\rho_{MV} - \sigma_M\sigma_Y\rho_{MY} - \sigma_V\sigma_Y\rho_{VY} + \sigma_Y^2]dt + [\sigma_M dZ_M + \sigma_V dZ_V - \sigma_Y dZ_Y]. \quad (7)$$

Recall that all terms in (7) are random variables. When we write μ_M we actually denote a random variable $\mu_M(t, \omega)$: $[0, T] \times \Omega \to R$. This means that dP/P is a random variable and one can compute its expectation and variance. These are given by

$$E\left(\frac{dP}{P}\right) = E(\mu_M + \mu_V - \mu_Y + \sigma_M\sigma_V\rho_{MV} - \sigma_M\sigma_Y\rho_{MY} - \sigma_V\sigma_Y\rho_{VY} + \sigma_Y^2)dt; \quad (8)$$

$$\mathrm{Var}\left(\frac{dP}{P}\right) = (\sigma_M^2 + \sigma_V^2 + \sigma_Y^2 + 2\sigma_M\sigma_V\rho_{MV} - 2\sigma_M\sigma_Y\rho_{MY} - 2\sigma_V\sigma_Y\rho_{VY})dt \quad (9)$$

Equations (8) and (9) are both informative and intuitively clear. Equation (8) says that expected inflation is the sum of the expected growth in money supply and velocity minus the expected growth in output as well as the result of four more terms which we discuss in the next section. Equation (9) relates the variance of inflation to the three variances σ_M^2, σ_V^2, σ_Y^2 and also to three other terms.

5. Discussion

Several remarks can be made about (7). First, this equation generalizes the deterministic quantity theory equation (3). To obtain (3) from (7) simply assume that the three diffusion coefficients are zero; that is, let $\sigma_M = \sigma_V = \sigma_Y = 0$. This tells us that the deterministic quantity theory of money ignores the existence of nonzero variances in the data concerning money supply, velocity of money, and real output.

Second, equation (7) identifies several terms which are important. These terms are $\sigma_M\sigma_V\rho_{MV}$, $\sigma_M\sigma_Y\rho_{MY}$, $\sigma_V\sigma_Y\rho_{VY}$ and σ_Y^2. The economic meaning of these terms is straightforward. Note that $\sigma_M\sigma_V\rho_{MV}$ denotes the covariance between the proportional rates of change of money supply and velocity; $\sigma_M\sigma_Y\rho_{MY}$ denotes the covariance between the proportional rates of change of money supply and real output; finally, $\sigma_V\sigma_Y\rho_{VY}$ denotes the covariance between the proportional rates of change of velocty and real output; σ_Y^2 denotes the variance of the proportional rate of change of real output. The three terms in the second bracket in (7) are the three sources of noise, dZ_M, dZ_V, dZ_Y, each multiplied by the standard deviations of the proportional rate of change of the money supply, velocity, and real output, respectively.

Third, equation (7), not only identifies several terms described above, but also specifies the exact relationship among these terms. Although economists recognize the importance of covariances between rates of growth of money supply and real output, no equation has appeared in the economic or econometric literature that has a specification such as (7). Some insights are occasionally obtained about the various covariances in (7) from simple single-equation econometric models, but such models are not as complete as (7). For example, one can regress linearly dP/P on dM/M and dY/Y and obtain some estimates of covariances, but such a model would be a special case of (7).

Fourth, because of the tautological nature of the quantity theory of money, (7) gives a rather exhaustive list of factors that can affect dP/P. Such a list of factors can be of great importance both theoretically and empirically. Theoretically, the interest in (4)–(6) is motivated by the goal of obtaining general results. Empirically, the computation of $E(dP/P)$ and $\mathrm{Var}(dP/P)$ can be expected to be more interesting in a general model than in a specialized one.

Fifth, equations (7)–(9) remain tautological. Lucas (1980) has suggested that some theoretical models, such as Sidrauski (1967a, 1967b) or Tobin (1965), have been developed to illustrate the dependence of inflation on the growth of money supply. Therefore, the deterministic quantity theory of money and its implications have some theoretical coherence. Aspects of the present stochastic version of the quantity theory of money could be supported by Gertler and Grinols (1982) and more recently Den Haan (1990) and Grinols and Turnovsky (1993). More specifically, Grinols and Turnovsky (1993) develop a three-agent model with continuous time stochastic variables, uncertain private production, uncertain government expenditures, and forward-looking rational agents. Although Grinols and Turnovsky (1993) wish to study the role of risk in a stochastic macroeconomic model with a financial sector, they confirm that an increase in the growth rate of money supply increases inflation and also that increased uncertainty in monetary growth rates also affects the variablity of inflation. Thus their results support our equations (8) and (9).

6. Examples

We now use two annual U.S. data sets to numerically decompose inflation and its volatility. The first data set, obtained from Gordon (1990), uses annual rates of growth of real GNP, the implicit price deflator M1, and velocity for the post-WWII period 1947–1988. To compute the expectation of U.S. inflation and its variance from (8) and (9), we need numerical expressions for the three Itô processes describing money supply, velocity, and real GNP changes. In other words, we need estimates for the six functions μ_M, σ_M, μ_V, σ_V, μ_Y, and σ_Y in (4)–(6). From equations (8) and (9) we also observe that we need estimates for the three correlation functions ρ_{MV}, ρ_{MY}, and ρ_{VY}. The simplest and most important case is when these six random functions are approximated by their sample annual means and standard deviations for the data used during the sample period 1947–1988. Straightforward calculations allow us to write

$$\frac{dM(t)}{M(t)} = .0478\,dt + .0303\,dZ_M(t) \tag{10}$$

$$\frac{dV(t)}{V(t)} = .0261\,dt + .0336\,dZ_V(t) \tag{11}$$

$$\frac{dY(t)}{Y(t)} = .0324\,dt + .0262\,dZ_Y(t) \tag{12}$$

as numerical expressions for (4)–(6). This is the most important case because we wish to contrast the traditional deterministic quantity theory of money, which uses constant sample means as explained in section 2, with the generalized stochastic version. This can be done when we compare the one deterministic equation (3) with the two stochastic equations (8) and (9). Note that (3) relates sample means which suggests that we also use sample means in (8).

The implication of (10)–(12) is that the level variables $M(t)$, $V(t)$, and $Y(t)$ follow stochastic processes of the form

$$M(t) = M(0)\exp\{[.0478 - 0.5(.0303)^2]t + .0303Z_M(t)\}$$

$$V(t) = V(0)\exp\{[.0216 - 0.5(.0336)^2]t + .0336Z_V(t)\}$$

$$Y(t) = Y(0)\exp\{[.0324 - 0.5(.0262)^2]t + .0262Z_Y(t)\}$$

where $M(0)$, $V(0)$, $Y(0)$ denote money supply, velocity, and real GNP for the initial year of 1947; t denotes time measured in years since 1947; and $Z_M(t)$, $Z_V(t)$, and $Z_Y(t)$ are standardized Wiener processes. These three last equations are generalizations of the simple deterministic equations that describe the usual exponential rate of growth of $M(t)$, $V(t)$, and $Y(t)$. Furthermore, when taking the natural logarithms of each of these last three equations we may conclude that such a natural logarithm is a random variable distributed normally with a certain mean and variance. For example

$$\ln \frac{M(t)}{M(0)} = \left[.0478 - \frac{1}{2}(.0303)^2\right] t + .0303Z_M(t)$$

has mean $[.0478 - 0.5(.0303)^2]t$ and variance $(.0303)^2 t$. Since the logarithm of $M(t)/M(0)$ is a normally distributed random variable we say that the money supply, velocity, and real output are lognormally distributed. For a detailed analysis of these issues, see Cox and Rubinstein (1985).

The next question to be addressed is the computation of the three correlation functions. Again, the instantaneous random correlations are approximated by their sample averages during 1947–1988 as follows: we work backward by computing the annual errors $dZ_M(t)$, $dZ_V(t)$, and $dZ_Y(t)$ from (10)–(12) and the actual corresponding rates. In other words, for each period t in our sample 1947–1988, given the actual rates $dM(t)/M(t)$, $dV(t)/V(t)$, $dY(t)/Y(t)$, and the approximations in (10)–(12), we solve for $dZ_M(t)$, $dZ_V(t)$, and $dZ_Y(t)$. Afterward we use the error sets $\{dZ_M(t), dZ_V(t), dZ_Y(t), t = 1948\text{–}1988\}$ to compute $\rho_{MV} = -.5770$, $\rho_{MY} = .1321$, and $\rho_{VY} = .2757$. Parenthetically, we also report that the error sets $\{dZ_M(t), dZ_V(t), dZ_Y(t)\}$ satisfy the required properties of a standardized Wiener process that $E(dZ_M) = E(dZ_V) = E(dZ_Y) = 0$ with all three variances being equal to 1.

With the above preliminaries completed, the U.S. rate of inflation and its variance during 1947–1988 can be decomposed as:

$$\begin{aligned}\frac{dP}{P} &= \mu_M + \mu_V - \mu_Y + \sigma_M\sigma_V\rho_{MV} - \sigma_M\sigma_Y\rho_{MY} - \sigma_V\sigma_Y\rho_{VY} + \sigma_Y^2 \\ &= (0.0478) + (0.0261) - (0.0324) + (0.0303)(0.0336)(-0.5770) \\ &\quad - (0.0303)(0.0262)(0.1321) - (0.0336)(0.0262)(0.2757) + (0.0262)^2 \\ &= 0.0413 \end{aligned} \tag{13}$$

$$\begin{aligned}\text{Var}\,\frac{dP}{P} &= \sigma_M^2 + \sigma_V^2 + \sigma_Y^2 + 2\sigma_M\sigma_V\rho_{MV} - 2\sigma_M\sigma_Y\rho_{MY} - 2\sigma_V\sigma_Y\rho_{VY} \\ &= (.0303)^2 + (.0336)^2 + (.0262)^2 + 2(.0303)(.0336)(-.5770) \\ &\quad - 2(.0303)(.0262)(.1321) - 2(.0336)(.0262)(.2757) \\ &= 0.00087 \end{aligned} \tag{14}$$

Note from (14) that inflation's standard deviation is given by $\{\text{Var}\ dP/P\}^{1/2} = .0295$, that is, inflation's annualized volatility during 1947–1988 is 2.95%.

If, instead of using (13) and (14) to decompose inflation and its variance we compute, from inflation data during 1947–1988, its average and standard deviation we obtain 4.15% and 2.52%, respectively. Obviously, the tautological nature of the quantity theory of money forces these numbers to be equal to (13) and (14), except for rounding off errors. However, analysis of (13) and (14) gives a much richer insight into inflation and its variance than do the two numbers 4.15% and 2.52%, because we have an explicit numerical expression of these numbers as sums and differences of specific terms.

To contrast the above results with another period we use next the long-run U.S. data collected and reported in Friedman and Schwartz (1982) for the period 1869–1947. In this sample M, Y, and P denote M2, national income, and the implicit price deflator.[1] Using the same procedure, the approximations of (4)–(6) are now given by

$$\frac{dM(t)}{M(t)} = .0607\,dt + .0681\,dZ_M(t) \tag{15}$$

$$\frac{dV(t)}{V(t)} = -.0196\,dt + .0725\,dZ_V(t) \tag{16}$$

$$\frac{dY(t)}{Y(t)} = .0330\,dt + .0727\,dZ_Y(t) \tag{17}$$

while $\rho_{MV} = .1304$, $\rho_{MY} = .5129$, $\rho_{VY} = .7531$.

In contrast to (13) and (14), inflation and its variance for the almost century-long data sample are decomposed below:

$$\begin{aligned}\frac{dP}{P} &= \mu_M + \mu_V - \mu_Y + \sigma_M\sigma_V\rho_{MV} - \sigma_M\sigma_Y\rho_{MY} - \sigma_V\sigma_Y\rho_{VY} + \sigma_Y^2 \\ &= (.0607) + (-.0196) - (.0330) + (.0681)(.0725)(.7531) \\ &\quad - (.0681)(.0727)(.5129) - (.0725)(.0727)(.7531) + (.0727)^2 \\ &= 0.001 \qquad (18)\end{aligned}$$

$$\begin{aligned}\text{Var}\,\frac{dP}{P} &= \sigma_M^2 + \sigma_V^2 + \sigma_Y^2 + 2\sigma_M\sigma_V\rho_{MV} - 2\sigma_M\sigma_Y\rho_{MY} - 2\sigma_V\sigma_Y\rho_{VY} \\ &= (.0681)^2 + (.0725)^2 + (.0727)^2 + 2(.0681)(.0725)(.1304) \\ &\quad - 2(.0681)(.0727)(.5129) - 2(.0725)(.0727)(.7531) \\ &= 0.003449 \qquad (19)\end{aligned}$$

From (19) we compute an annual inflation volatility of 5.87%. The actual numbers for inflation and its volatility obtained directly from annual inflation data during 1869–1947 are identically the same, i.e., 0.1% and 5.87%, respectively.

What insights can the decompositions of these two sample periods offer? First, actual inflation during 1947–1988 was much higher than actual inflation during 1869–1947; the respective numbers are 4.15% and 0.1%. Equations (12) and (17) tell us that real output during these two periods grew at a similar rate, i.e., 3.24% for 1947–1988 and 3.3% for 1869–1947. What then explains the substantial difference in inflation rates? Looking at money growth rates complicates the picture: Money grew at a slower rate during 1947–1988 than it did during 1869–1947, i.e., 4.78% in contrast to 6.07%. Intuitively, one would expect faster money growth to contribute to higher inflation when real output grows at the same rate. The decomposition shows that the velocity of money solves partially the puzzle: During 1869–1947, the velocity of money declined by 1.96% per year while it grew by 2.61% during 1947–1988.

The second and final insight concerns the volatility of inflation. Observe that it is much higher during 1869–1947 than during the post-WWII period, i.e., 5.87% compared to 2.52%, respectively. The deterministic quantity theory of money offers no explanations for such differences. An economist's intuition would suggest that higher inflation volatility may be generated by higher volatility of money growth and/or higher volatility of real output. Such an intuition is supported by the data. During 1869–1947, the volatilities of money growth and real output were 6.81% and 6.42%, compared to 3.03% and 2.62% respectively for 1947–1988. But without a specific model these volatilities cannot be put into an algebraic expression. Equations (14) and (19) show the exact decomposition of inflation volatilities as a sum of the volatilities of money growth, output growth, and velocity change with also, their three covariances.

7. Summary and conclusions

This paper addresses the important question of how inflation and its volatility can be decomposed into several terms to offer insights into the economic sources that generate inflation.

Such a decomposition requires an acceptable inflation model and this paper uses the quantity theory to extend its deterministic dynamic version into a continuous time dynamic stochastic version. This generalization is justified both by the actual behavior of the quantity theoretic variables and by the underlying methodological background of diffusion modeling. The generalization obtained yields two expressions that decompose both inflation and its variance into several, economically meaningful terms. Finally two examples illustrate the numerical decomposition and contrast the factors that explain U.S. inflation and its volatility.

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Notes

1. Those familiar with the Friedman and Schwartz (1982) book will recall that their data cover the period 1869–1977. We thank two anonymous referees who suggested that we only use 1869–1947 to avoid an overlap with our first example and thus to sharpen the contrast between the two illustrations.

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